



Calhoun: The NPS Institutional Archive
DSpace Repository

Theses and Dissertations

1. Thesis and Dissertation Collection, all items

1993-06

Out of plane solutions of submarines in free positive buoyancy ascent

Aydin, Ibrahim

Monterey, California. Naval Postgraduate School

<http://hdl.handle.net/10945/27195>

Copyright is reserved by the copyright owner

Downloaded from NPS Archive: Calhoun



Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

<http://www.nps.edu/library>

DU.
NA
MC

LIBRARY
GRADUATE SCHOOL
1945-5101

Approved for public release; distribution is unlimited.

**Out of Plane Solutions of Submarines
in Free Positive Buoyancy Ascent**

by

Ibrahim Aydin

//

Lieutenant Junior Grade, Turkish Navy
B.S., Turkish Naval Academy, 1987

Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL

June, 1993

REPORT DOCUMENTATION PAGE

REPORT SECURITY CLASSIFICATION UNCLASSIFIED		1b. RESTRICTIVE MARKINGS	
SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited	
DECLASSIFICATION/DOWNGRADING SCHEDULE			
PERFORMING ORGANIZATION REPORT NUMBER(S)		5. MONITORING ORGANIZATION REPORT NUMBER(S)	
NAME OF PERFORMING ORGANIZATION Naval Postgraduate School		7a. NAME OF MONITORING ORGANIZATION Naval Postgraduate School	
6b. OFFICE SYMBOL (if applicable) ME			
ADDRESS (City, State, and ZIP Code) Monterey, CA 93943-5000		7b. ADDRESS (City, State, and ZIP Code) Monterey, CA 93943-5000	
NAME OF FUNDING/SPONSORING ORGANIZATION		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
6b. OFFICE SYMBOL (if applicable)			
ADDRESS (City, State, and ZIP Code)		10. SOURCE OF FUNDING NUMBERS	
		PROGRAM ELEMENT NO.	PROJECT NO.
		TASK NO.	WORK UNIT ACCESSION NO.
TITLE (Include Security Classification) Stability of Plane Solutions of Submarines in Free Positive Buoyancy Ascent (Unclassified)			
PERSONAL AUTHOR(S) Fotis A. Papoulas			
TYPE OF REPORT Master's Thesis		13b. TIME COVERED FROM 09/92 TO 06/93	14. DATE OF REPORT (Year, Month, Day) June 1993
15. PAGE COUNT 143			
SUPPLEMENTARY NOTATION The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the United States Government.			
COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
ELD	GROUP	Submarine Stability, Continuation, Bifurcation, Pitchfork, Hysteresis, Cusp	
ABSTRACT (Continue on reverse if necessary and identify by block number) This thesis analyzes the problem of motion stability of submarines in free positive buoyancy ascent under equality conditions such as control surface jam and loss of propulsion system response. We employ fully nonlinear coupled six degree of freedom equations of motion and we allow response to occur in combined vertical and horizontal planes. Continuation and homotopy theory techniques are utilized to trace all possible steady state solutions in six degrees of freedom, while local perturbation reveals their stability properties. Vehicle geometric properties and control surface deflections are used as primary bifurcation parameters. Regions in parameter space are identified where extreme sensitivity of solutions to geometric properties and hydrodynamic modeling is present.			
DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS		21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED	
NAME OF RESPONSIBLE INDIVIDUAL Professor Fotis A. Papoulas		22b. TELEPHONE (Include Area Code) (408) 646-3381	22c. OFFICE SYMBOL Code ME/Pa

ABSTRACT

This thesis analyzes the problem of motion stability of submarines in free positive buoyancy ascent under casualty conditions such as control surface jam and loss of propulsion system response. We employ fully nonlinear, coupled six degree of freedom equations of motion and we allow response to occur in combined vertical and horizontal planes. Continuation and homotopy theory techniques are utilized to trace all possible steady state solutions in six degrees of freedom, while local perturbation reveals their stability properties. Vehicle geometric properties and control surface deflections are used as primary bifurcation parameters. Regions in parameter spaces are identified where extreme sensitivity of solutions to geometric properties and hydrodynamic modeling is present.

TABLE OF CONTENTS

I. INTRODUCTION	1
II. PROBLEM FORMULATION	7
A. PROBLEM STATEMENT	7
B. EQUATIONS OF MOTION	8
1. Dynamic Variables	16
2. Mass Distribution Parameters	16
3. Remaining Parameters	17
C. STEADY STATE CONDITIONS	17
D. PRINCIPLES OF CONTINUATION	19
III. CONTINUATION IN δs	22
A. GENERAL	22
B. PITCHFORK BIFURCATION	23
C. CONDITIONS	29
1. Defining Additional Terms	29
a. Excess Buoyancy δB	29
b. Longitudinal Center of Buoyancy , x_{GB}	30

c.	Vertical Center of Buoyancy, z_{GB}	30
2.	Assumed Conditions	30
a.	Lateral Centers of Gravity, y_G , and Buoyancy, y_B	30
b.	Propeller Speed , n (revolution per minute)	30
c.	Propeller Coefficients , K_{prop} and N_{prop}	30
d.	Vertical Center of Buoyancy, z_{GB}	30
D.	STEADY STATE RESULTS	31
IV.	CONTINUATION IN δr	38
A.	GENERAL	38
B.	HYSTERESIS PHENOMENON	38
C.	STEADY STATE RESULTS	42
V.	CONTINUATION IN x_{GB}	49
A.	GENERAL	49
B.	STEADY STATE RESULTS	49
1.	$\delta r = 0$ Degrees, δs is Perturbation Parameter	49
2.	$\delta s = -20$ Degrees, δr is the Perturbation Parameter	54
3.	$\delta s = 0$ Degrees, δr is the Perturbation Parameter	59
VI.	CONTINUATION IN δB	64
A.	GENERAL	64

B.	STEADY STATE RESULTS	65
1.	$\delta r=0$ Degrees, δs is the Perturbation Parameter	65
2.	$\delta s=-20$ Degrees, δr is the Perturbation Parameter	71
3.	$\delta s=0$ Degrees, δr is Perturbation Parameter	76
VII.	CONTINUATION IN z_{GB}	81
A.	GENERAL	81
B.	STEADY STATE RESULTS	81
1.	$\delta r= 0$ Degrees, δs is the Perturbation Parameter	81
2.	$\delta s=-20$ Degrees, δr is the Perturbation Parameter	86
3.	$\delta s = 0$ Degrees, δr is the Perturbation Parameter	91
VIII.	CUSP CURVES	96
A.	GENERAL	96
B.	RESULTS	101
IX.	DYNAMIC STABILITY ANALYSIS	109
A.	GENERAL	109
B.	DYNAMIC STABILITY RESULTS	109
C.	NUMERICAL INTEGRATION RESULTS	112
X.	CONCLUSIONS AND RECOMMENDATIONS	118

APPENDIX	119
1. BIFPACK	120
2. MAIN PROGRAM	121
3. SUBROUTINE FCN.	122
4. SAMPLE RUN	127
LIST OF REFERENCES	131
INITIAL DISTRIBUTION LIST	133

ACKNOWLEDGEMENTS

It is with deep gratification I would like to thank Dr. Fotis A. Papoulias for his guidance, encouragement, patience and support throughout this research.

Finally, and most importantly, I would like to thank my wife Zehra, for her unending support and encouragement throughout my graduate education and thesis research.

I. INTRODUCTION

The dynamic response of a submarine under casualty conditions constitutes a crucial, and frequently limiting, factor in establishing the vehicle's submerged operating envelope. As basic casualty conditions in the context of this work, we mean the loss of control surface and/or propulsion system response, and a flooding casualty where the boat must either be brought to the surface or stabilized to a new operating depth. Of particular significance is the study of the ability of the boat to recover from a control surface jam. There exist several factors which determine the severity of such a situation as well as the recovery procedures; the initial conditions (speed, depth, pitch angle, etc.) during the jam, the actual control surface angles, reversing time and backing power of the propulsion system, the ability to blow ballast, and the time between recognition of casualty and initiation of proper recovery procedures. To this end, it is crucial that we have a clear understanding of the dynamics of the boat during an emergency situation.

The traditional methods for establishing dynamic stability of motion concentrate mainly on eigenvalue analysis during small perturbations around nominal straight line paths Clayton and Bishop [Ref. 1]. Two indices are utilized, a stability index G_v for the vertical plane and G_h for the horizontal plane Roddy [Ref. 2]. In terms of the slow motion derivatives, these indices are given by:

$$G_v = 1 - \frac{M_w (Z_q + m)}{Z_w (m_q - x_G m)} ,$$

$$G_h = 1 - \frac{N_v (Y_r - m)}{Y_v (N_r - x_G m)} .$$

Positive values for these indices, whose usage dates back to the 1950's, indicates motion stability in the corresponding plane. The underlying assumption in these criteria is that during normal straight line motions, the coupling between horizontal and vertical plane motions is relatively weak, and can be neglected. Although vortex shedding and flow separation introduces a certain degree of coupling, the above assumption has been proven quite useful in design and analysis. However, for a high-speed fast-maneuvering submarine operating at the extremes of her submerged operating envelope, or during emergency situations, the above assumption of uncoupled motions breaks down. High amplitude motions may take place in all six degrees of freedom, and the nonlinear interactions between the various modes of motion become more pronounced. Therefore, we have to carefully consider the motion characteristics allowing for coupling between horizontal and vertical planes.

The implications of nonlinear effects and coupling are numerous. In the case of roll motion, which is one of the most critical responses, there is growing evidence of complicated dynamics and chaotic response under certain excitations Falzarano, Shaw and Troesch [Ref. 3], Taz Ul Mulk and Falzarano [Ref. 4].

The motion of a submarine in the ocean environment involves some of the most complex fluid-structure interaction problems. For example, the steady and unsteady characteristics of the stern, and especially with regard to the effect of the wake and rolled-up body vortices on propulsor unsteadiness, and consequences for noise and hull vibrations are important problems of great concern. At present, too little is known in depth about the flow at the stern of ships and submarines and the footprints of their ensuing wakes in homogenous and stratified media Sarpkaya [Ref. 5].

Typical vortex structure about a submerged body can be seen in Figure 1, which is taken from Lugt [Ref. 6].

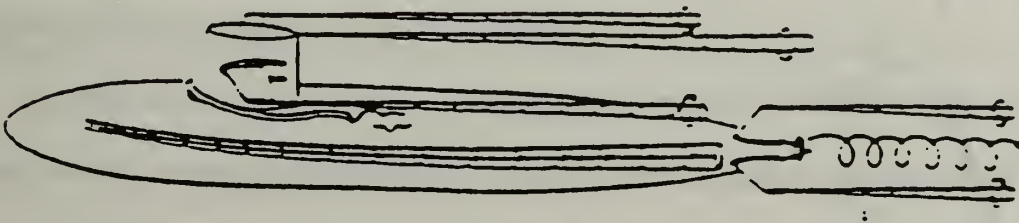


Figure 1. Typical Vortex Structure About a Submerged Body (From Lugt, 1981).

When an axisymmetric body moves at a sufficiently large angle of attack, the boundary layer vorticity may lift off the surface and sheets of vorticity are convected downstream while rolling up into streamwise vortices on the leeward

side of the body. This separation and roll-up phenomenon, known as the crossplane separation, occurs on almost all maneuvering bodies at sufficiently high angles of attack. The roll-up of the vortices and their subsequent evolution are extremely important for the proper design of a submerged body and more important for the determination of the interaction of the vortices with the stern and the propulsion system. It is this interaction that determines the maneuverability and control of a submerged body Sarpkaya [Ref. 5].

In the case of submarine motions, there is evidence of bifurcation phenomena and extreme sensitivity of response to initial conditions and control actions during emergency

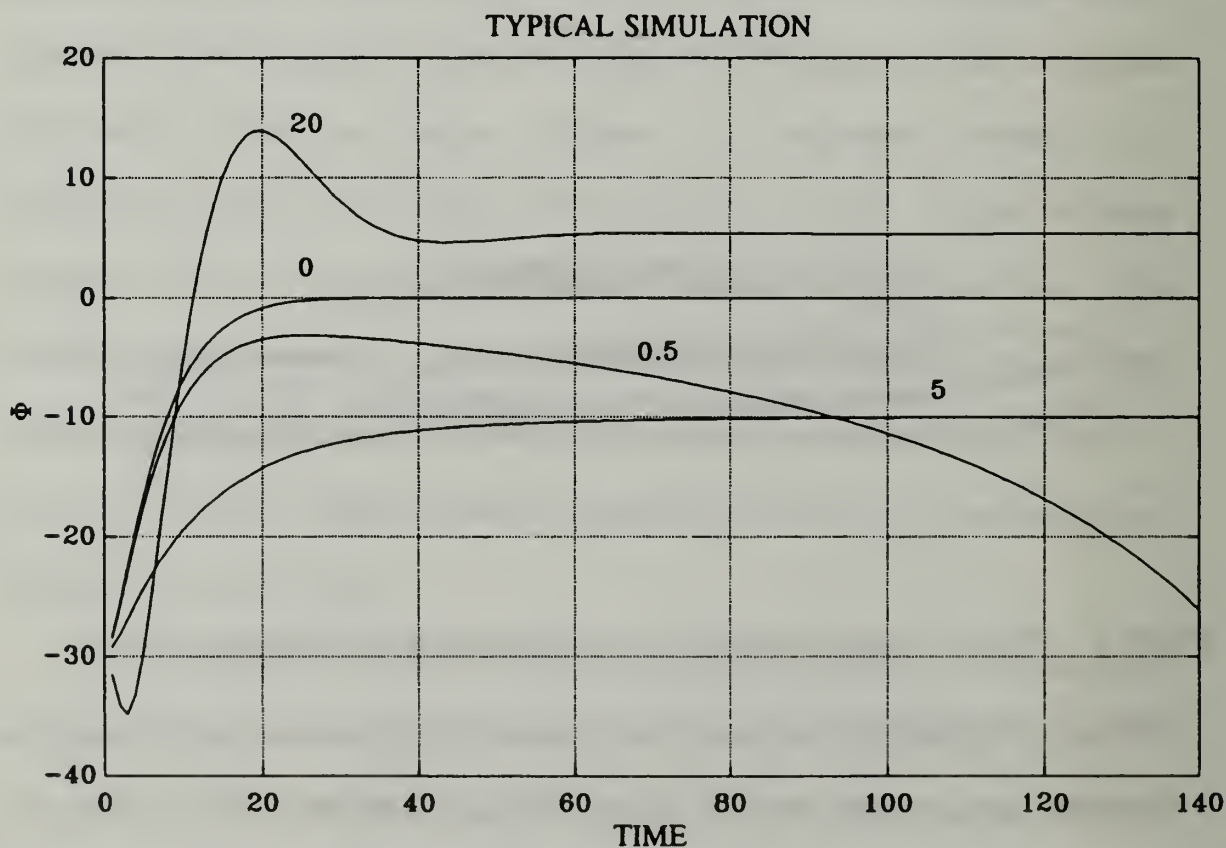


Figure 2. Typical Simulation Results.

ascent scenarios such as recovery from a dive plane jam. As motivation for the analysis that follows we present typical simulation results in Figure 2. The time simulation is in terms of vehicle roll angle Φ versus time for 2% excess buoyancy with the buoyancy force located 1% of the vehicle length forward of the center of gravity, -6 degrees dive plane angle, and 0.1 feet metacentric height. Four different recovery actions are shown, all parametrized by the applied rudder angle in degrees. It can be seen that although zero rudder angle appears to bring the vehicle roll angle back to zero in the shortest time, the response does not persist. Small nonzero values for the rudder angle develop excessive roll angles in a divergent way, while larger rudder angles reduce the amount of roll. Clearly the vehicle effective rudder angle is largely affected by the amount of vorticity and currents in the flow field, and its actual value can not be known exactly. Since situations like the one presented in the graph should be avoided, we need to develop a mechanism for assessing those regions in the parameter space where the response can not be simulated with confidence.

Stability in emergency ascent has been studied in the vertical plane. In Booth [Ref. 7] and [Ref. 8], the vehicle response was distinguished into either a nearly vertical ascent or predominantly forward motion. In Papoulias and McKinley [Ref. 9], it was found that the above distinction is not always meaningful as a result of the many parameters that affect the problem. This latter study maintained some vertical plane restrictions, although it indicated the potential existence of pitchfork bifurcations which led to coupled out-of-plane solutions. In this work, we relax the requirement for vertical plane motions, and we analyze the stability properties of all possible steady states in six degrees of freedom.

We employ a combination of singularity theory Golubitsky and Schaeffer [Ref. 10], bifurcation theory Guckenheimer and Holmes [Ref. 11], and numerical continuation methods Seydel [Ref. 12] in order to capture all steady state solutions that are physically admitted by the coupled nonlinear equations of motion. The primary bifurcation parameters used are: amount and location of excess buoyancy, diveplane and rudder deflection, and the metacentric height. Solution branching is shown to occur in various forms, including single and multiply connected pitchfork bifurcations, separation of solutions, hysteresis, and teardrop branches. Dynamic loss of stability in the form of Poincare-Andronov-Hopf bifurcations to periodic solutions is also identified. It is shown that for certain ranges of parameters, these periodic solutions persist in the vicinity of inverted pendulum steady states. Finally, we summarize our results in the form of bifurcation graphs which identify parameter regions with qualitatively different asymptotic response characteristics. For demonstration purposes, all computations in this work are performed for the Swimmer Delivery Vehicle, a 17.4 feet vehicle, for which a complete set of hydrodynamic and geometric properties is available Smith, Crane and Summey [Ref. 13]. Unless otherwise specified, all results in this work are presented in dimensional form, linear dimensions in feet, velocities in ft/sec, angular deflections in degrees, and time in seconds.

II. PROBLEM FORMULATION

A. PROBLEM STATEMENT

Controlling emergency ascent situations on submersible vehicles such as dive plane jam recovery is of concern to the submariners. In order to control such situations, one must first be able to predict the dynamic response of positively buoyant submersibles.

Dynamic response equations of motion describe the maneuvering characteristics of submersible vehicles for six degrees of freedom. These equations assume constant coefficients for hydrodynamic forces and moments approximated by zero frequency added mass and damping terms plus the quadratic terms for drag forces. The constant coefficients vary for each vehicle and depend on such things as vehicle body shape, location and magnitude of vehicle buoyancy, position of bow and stern planes, position of rudder, vehicle speed, vehicle mass characteristics, vehicle hydrodynamic coefficients, propeller rpm and control surface inputs. This thesis uses the equations of motion and hydrodynamic coefficients for a submerged Mark IX Swimmer Delivery vehicle (SDV) developed by Smith, Crane and Summey [Ref. 13] to forecast the dynamic behavior of submersibles in a positively buoyant condition.

For submarines where high amplitude motions may take place in all six degrees of freedom, nonlinear interactions between the various modes of motion may become more pronounced. In particular, there is growing evidence of bifurcation phenomena during high speed maneuvering and emergency ascent scenarios such as recovery from a dive

plane jam. In these cases, it is no longer true that decoupled linear analysis techniques are sufficient and one is forced to consider the true character of six degrees of freedom motions. So, in this thesis, six degrees of freedom equations of motions have been used to investigate branching behaviors of the equations of motions. This is done under the condition of changing certain system parameters while keeping others constant.

B. EQUATIONS OF MOTION

The equations of motion given below are referenced to a right-hand orthogonal axis system fixed in the body as shown in Figure 3. The origin of this body axis system is located in the vehicle's X-Z plane of symmetry on the mid-body centerline behind the vehicle's nose. Positive directions for control surface deflections, forces and angular moments, and linear and angular velocity components are shown in Figure 3. In the following equations, differentiation with respect to time is denoted by a dot over a quantity. The six degrees of freedom equations of motion for a submarine in surge, sway, heave, roll, pitch, and yaw, respectively, are:

$$m \times [\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = X_H + X_W + X_C \quad (1)$$

$$m \times [\dot{v} + ur - wp + x_G(pq + \dot{r}) - y_G(p^2 + r^2) + z_G(qr - \dot{p})] = Y_H + Y_W + Y_C \quad (2)$$

$$m \times [\dot{w} - uq + vp + x_G(pr - \dot{q}) + y_G(qr + \dot{p}) - z_G(p^2 + q^2)] = Z_H + Z_W + Z_C \quad (3)$$

$$I_x \dot{p} + (I_z - I_y)qr + I_{xy}(pr - \dot{q}) - I_{yz}(q^2 - r^2) - I_{xz}(pq + \dot{r}) + m \times [y_G(\dot{w} - uq + vp) - z_G(\dot{v} + ur - wp)] = K_H + K_W + K_C \quad (4)$$

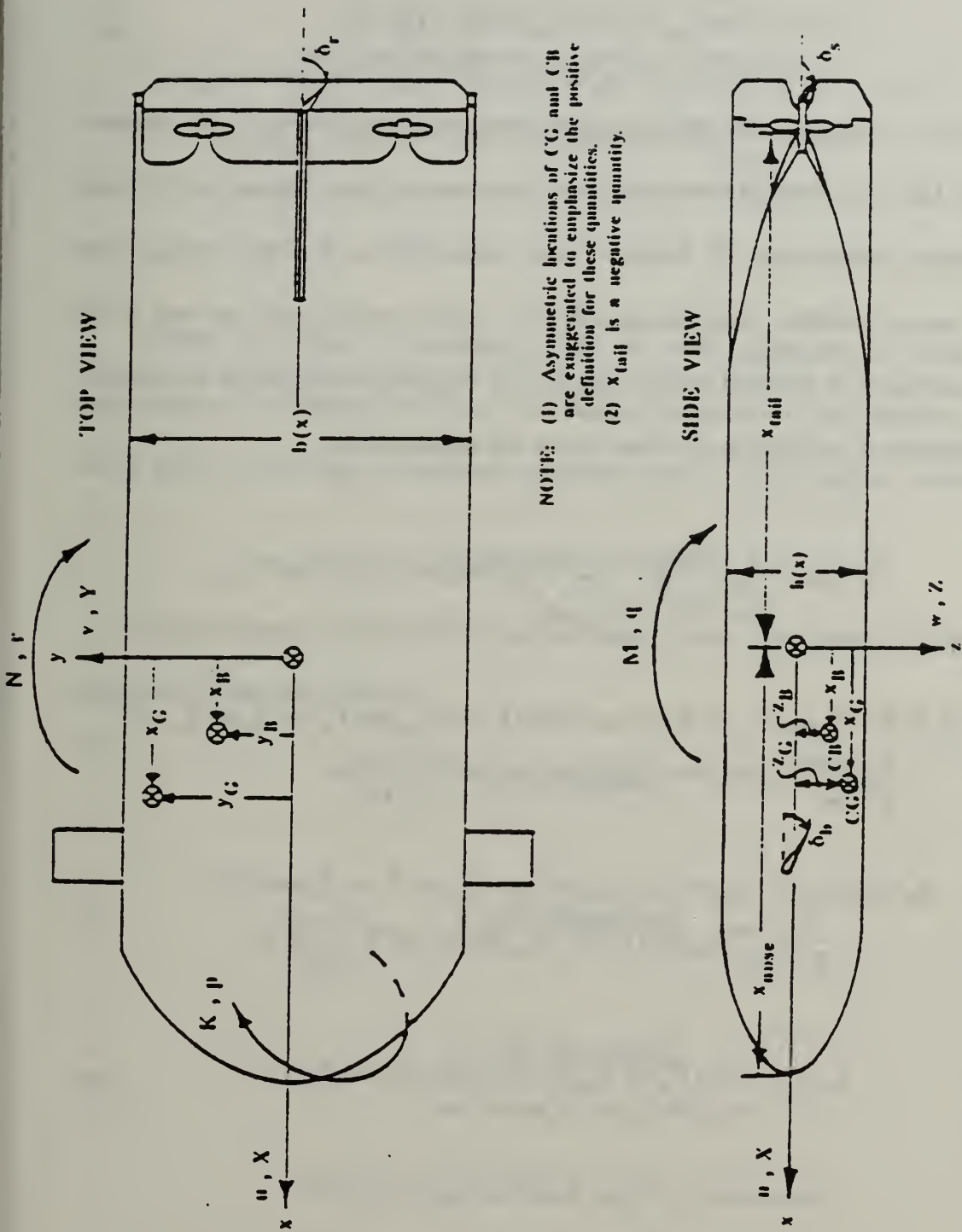


Figure 3. Schematic Indicating Positive Directions of Axes, Angles, Velocities, Forces, and Moments

$$I_y \dot{q} + (I_x - I_z) p r - I_{xy} (q r + \dot{p}) + I_{yz} (p q - \dot{r}) + I_{xz} (p^2 - r^2) - m \times [x_G (\dot{w} - u q + v p) - z_G (\dot{u} - v r + w q)] = M_H + M_W + M_C \quad (5)$$

$$I_z \dot{r} + (I_y - I_x) p q - I_{xy} (p^2 - q^2) - I_{yz} (p r + \dot{q}) + I_{xz} (q r - \dot{p}) + m \times [x_G (\dot{v} + u r - w p) - y_G (\dot{u} - v r + w q)] = N_H + N_W + N_C \quad (6)$$

In these equations, the left hand sides represent inertial forces and moments (Newton's Law) and the right hand sides model the external forces. Subscript H reflects hydrodynamic contributions, W buoyancy and weight effects, C forces arising from control surface (rudders, dive planes, and bow planes) actions, and the rest of the symbols are based on standard notation and will be explained at the end of this section. The hydrodynamic radiation and viscous forces are expressed as:

$$X_H = X_{pp} p^2 + X_{qq} q^2 + X_{rr} r^2 + X_{pr} p r + X_{\dot{u}} \dot{u} + X_{wq} w q + X_{vp} v p + X_{vr} v r + X_{vv} v^2 + X_{ww} w^2 - C_{D0} u^2 \quad (7)$$

$$Y_H = Y_{\dot{p}} \dot{p} + Y_{\dot{r}} \dot{r} + Y_{pq} p q + Y_{qr} q r + Y_{\dot{v}} \dot{v} + Y_p u p + Y_r u r + Y_{vq} v q + Y_{wp} w p + Y_{wr} w r + Y_{uv} u v + Y_{vw} v w - \frac{1}{2} \rho \int_{x_{tail}}^{x_{nose}} [C_{D_Y} h(x) (v + x r)^2 + C_{D_Z} b(x) (w - x q)^2] \frac{(v + x r)}{U_{cf}(x)} dx \quad (8)$$

$$Z_H = Z_{\dot{q}} \dot{q} + Z_{pp} p^2 + Z_{pr} p r + Z_{rr} r^2 + Z_{\dot{w}} \dot{w} + Z_q u q + Z_{vp} v p + Z_{vr} v r + Z_{uw} u w + Z_{vv} v^2 - \frac{1}{2} \rho \int_{x_{tail}}^{x_{nose}} [C_{D_Y} h(x) (v + x r)^2 + C_{D_Z} b(x) (w - x q)^2] \frac{(w - x q)}{U_{cf}(x)} dx \quad (9)$$

$$K_H = K_{\dot{p}} \dot{p} + K_{\dot{r}} \dot{r} + K_{pq} p q + K_{qr} q r + K_{\dot{v}} \dot{v} + K_p u p + K_r u r + K_{vq} v q + K_{wp} w p + K_{wr} w r + K_{uv} u v + K_{vw} v w \quad (10)$$

$$M_H = M_{\dot{q}} \dot{q} + M_{pp} p^2 + M_{pr} pr + M_{rr} r^2 + M_{\dot{w}} \dot{w} + M_{q\dot{q}} uq + M_{vp} vp + M_{vr} vr + M_{w\dot{w}} uw + M_{vv} v^2 + \frac{1}{2} \rho \int_{x_{tail}}^{x_{nose}} [C_{D_Y} h(x)(v+xr)^2 + C_{D_Z} b(x)(w-xq)^2] \frac{(w-xq)}{U_{cf}(x)} x dx \quad (11)$$

$$N_H = N_{\dot{p}} \dot{p} + N_{\dot{r}} \dot{r} + N_{pq} pq + N_{qr} qr + N_{\dot{v}} \dot{v} + N_{p\dot{p}} up + N_{r\dot{r}} ur + N_{vq} vq + N_{wp} wp + N_{wr} wr + N_{v\dot{v}} uv + N_{vw} vw - \frac{1}{2} \rho \int_{x_{tail}}^{x_{nose}} [C_{D_Y} h(x)(v+xr)^2 + C_{D_Z} b(x)(w-xq)^2] \frac{(v+xr)}{U_{cf}(x)} x dx \quad (12)$$

These are given in customary form of series expansion in terms of the hydrodynamic coefficients and cross flow integral terms which are integrated over the entire length of the body and represent quadratic forces. The cross flow velocity U_{cf} is:

$$U_{cf} = [(v+xr)^2 + (w-xq)^2]^{1/2} \quad (13)$$

Hydrodynamic restoring forces and moments are due to the vehicle weight W and buoyancy B , and are given by:

$$X_W = -(W-B)\sin\theta \quad (14)$$

$$Y_W = (W-B)\cos\theta\sin\phi \quad (15)$$

$$Z_W = (W-B)\cos\theta\cos\phi \quad (16)$$

$$K_W = (y_G W - y_B B)\cos\theta\cos\phi - (z_G W - z_B B)\cos\theta\sin\phi \quad (17)$$

$$M_W = -(x_G W - x_B B) \cos \theta \cos \phi - (z_G W - z_B B) \sin \theta \quad (18)$$

$$N_W = (x_G W - x_B B) \cos \theta \sin \phi + (y_G W - y_B B) \sin \theta \quad (19)$$

Forces and moments, due to control surface deflections, are reflected as added drag in surge, while in sway, heave, pitch, and yaw they are directly proportional to the control surface deflections.

$$X_C = uq(X_{q\delta_s} \delta_s + X_{q\delta_b} \delta_b) + X_{r\delta_r} ur \delta_r + X_{v\delta_r} uv \delta_r + uw(X_{v\delta_s} \delta_s + X_{w\delta_b} \delta_b) + u^2(X_{\delta_s\delta_s} \delta_s^2 + X_{\delta_b\delta_b} \delta_b^2 + X_{\delta_r\delta_r} \delta_r^2) \quad (20)$$

$$Y_C = Y_{\delta_r} u^2 \delta_r \quad (21)$$

$$Z_C = u^2(Z_{\delta_s} \delta_s + Z_{\delta_b} \delta_b) \quad (22)$$

$$K_C = 0 \quad (23)$$

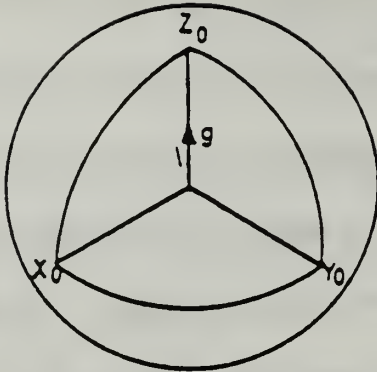
$$M_C = u^2(M_{\delta_s} \delta_s + M_{\delta_b} \delta_b) \quad (24)$$

$$N_C = N_{\delta_r} u^2 \delta_r \quad (25)$$

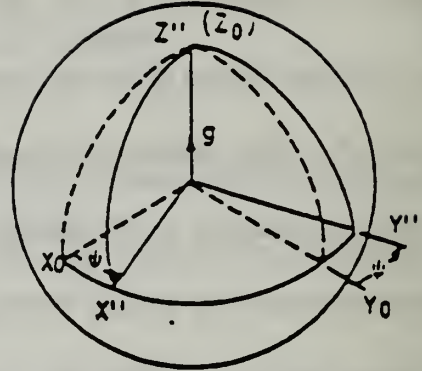
Usually, control surface deflections are kept intentionally small, and the linearity assumption in Equations (21), (22), (24), and (25) remains valid. Unlike the surface ship case, the roll moment K_C is zero for a submersible since the rudder is centered with the vehicle center plane. The hydrodynamic coefficients in equations (7) through (12) are functions of the frequency of motion, or what amounts to the same thing functions of

the maneuver at hand. In this work slowly varying reference motions are studied and therefore, it is assumed that they remain constant and equal to their zero frequency limit. It should be emphasized though, that the constant coefficient assumption would break down in studies related to the fast motions under the action of first order wave forces in the case of a nearly surfaced submarine. Another important assumption in this study is that they are assumed to be constant throughout the range of vehicle angle of attack. Ordinary maneuvering models are usually validated for angles of attack ± 15 degrees. For higher angles of attack, the cross flow drag terms C_{DY} and C_{DZ} dominate the response, and they are functions of the side and angle of attack. Considering them to be constant does not alter significantly the behavioral characteristics and qualitative bifurcation results that are derived. Similarly, C_{DY} and C_{DZ} are functions of speed due to the Reynolds number effect on cross flow drag. This is more pronounced in small size unmanned untethered vehicles, whereas for submarines the cross flow drag terms remain relatively constant over the entire speed range.

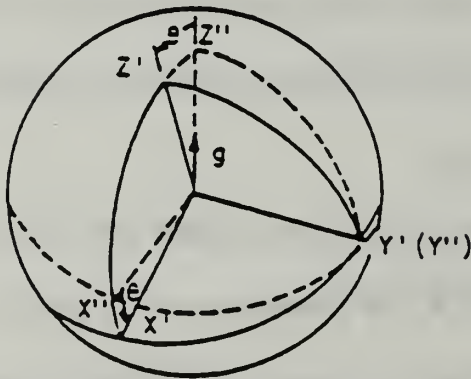
Since the equations of motions are referred to an axis system that is fixed in the vehicle, and thus translates and rotates with it, the orientation and the position of the moving body axis system relative to fixed inertial reference system must be specified. The orientation of the body axis system with respect to the inertial reference system is defined by the standard **Euler Angles** Ψ (yaw), Θ (pitch), and Φ (roll). The rotation sequence from the inertial reference system to the body axis system is Ψ , Θ , Φ as shown in Figure 4. So to complete the model, we need the expressions for the **Euler Angle** rates of change;



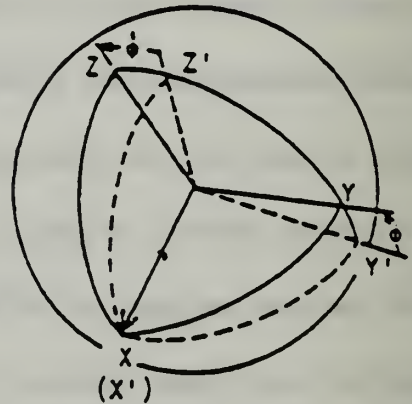
(1) Vehicle-Centered Gravity-Directed System parallel to inertial axis system.



(2) Horizontal Flight Reference System derived from $X_0Y_0Z_0$ by rotation about Z through yaw angle ψ .



(3) Roll-Resolved Flight Reference System derived from $X''Y''Z''$ by rotation about Y' through pitch angle θ .



(4) Vehicle Body Axis Reference System derived from $X'Y'Z'$ by rotation about X' through roll angle ϕ .

Figure 4. Unit Sphere Development of Euler Angles

$$\dot{\phi} = p + q \sin\phi \tan\theta + r \cos\phi \tan\theta \quad (26)$$

$$\dot{\theta} = q \cos\phi - r \sin\phi \quad (27)$$

$$\dot{\psi} = q \frac{\sin\phi}{\cos\theta} + r \frac{\cos\phi}{\cos\theta} \quad (28)$$

Finally, it is assumed that propulsion is inoperative and the propeller is rotating freely. Therefore, propulsive forces are not included in Equations (1) through (6). The driving mechanism for the vehicle is its excess buoyancy, $B-W > 0$. The problem is then to assess the asymptotic dynamic characteristics of the system during this condition of free positive buoyancy ascent.

The model presented above can be written in its state space form by selecting as state variables;

$$\begin{aligned} x_1 = u, \quad x_2 = v, \quad x_3 = w, \quad x_4 = p \\ x_5 = q, \quad x_6 = r, \quad x_7 = \phi, \quad x_8 = \theta \end{aligned} \quad (29)$$

where the first six describe the system motion and the last two its geometry. Notice that the yaw angle Ψ does not affect the equations of motion and is, therefore, not included in (29). Angle Ψ can be computed from (28), once the time histories of the state variables have been obtained.

In a compact vector form the state equations can be written as,

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \quad (30)$$

where bold face indicates a vector.

Major variables and parameters as defined by Smith, Crane, and Summey

[Ref. 13] are given below:

1. Dynamic Variables

- | | |
|-----------|--|
| u, v, w | - Linear velocity components of vehicle with respect to origin of body axes system relative to fluid. |
| p, q, r | - Angular velocity components of vehicle with respect to body axes system relative to inertial reference system. |
| X, Y, Z | - Hydrodynamic force components along body axes. |
| K, M, N | - Hydrodynamic moment components along body axes. |

2. Mass Distribution Parameters

- | | |
|-----------------|---|
| m | - Mass of the flooded vehicle, including the mass of the entrained fluid. |
| W | - Weight of the flooded vehicle, including the weight of the entrained fluid. ($=gm$; where g is the acceleration of gravity). |
| V | - Displacement volume of the vehicle. |
| B | - Buoyancy force acting on the vehicle ($\rho g V$). This is independent of the inertial mass distribution of the submersible vehicle, including whether or not it is flooded. |
| X_G, Y_G, Z_G | - Coordinates of the CG (center of gravity) in the body axis system (Figure 3). These will depend on the mass distribution of the vehicle, including the mass of the entrained fluid. |
| X_B, Y_B, Z_B | - Coordinates of the (center of buoyancy) in the body axis system (Figure 3). These are independent of the mass |

distribution system, but may vary with the addition or removal of external appendages.

I_x, I_y, I_z - Moments of inertia about the body system axes, including the entrained fluid.

I_{xy}, I_{xz}, I_{yz} - Products of inertia about the body system axes, including the entrained fluid.

3. Remaining Parameters

ρ - Mass density of fluid medium.

$b(x), h(x)$ - Width and height of vehicle in its xy and xz planes, respectively, at location x measured in the body axes system (Figure 3). These quantities are required in the integral defining the cross flow forces and moments in the equations of motion.

x_{nose}, x_{tail} - Coordinates of vehicle nose and tail as measured in body axis system. (Figure 3).

$\delta_s, \delta_b, \delta_r$ - Sternplane, bowplane and rudder deflection angles in radians (Figure 3).

C. STEADY STATE CONDITIONS

Steady state conditions are achieved when the submersible reaches constant linear and angular velocities which must assume finite values. Therefore, the body fixed linear accelerations and body fixed angular accelerations will be zero. Likewise, the vehicle will have reached constant angles of roll ϕ and pitch θ , making the derivatives of them equal to zero. When these values are put into Equations (1) through (6), and substituting

$$\dot{\phi} = \dot{\theta} = 0$$

in Equations (26) through (28) the steady state values of the angular velocities can be found as:

$$p = -\dot{\psi} \sin\theta \quad (31)$$

$$q = \dot{\psi} \sin\phi \cos\theta \quad (32)$$

$$r = \dot{\psi} \cos\phi \cos\theta \quad (33)$$

Combining Equations (31), (32), (33) with the equations (1) through (6) yields a system of nine coupled nonlinear algebraic equations in the form

$$f(\bar{\mathbf{x}}) = \mathbf{0} \quad (34)$$

where the overbar denotes an equilibrium solution.

Equation system (34) can be solved for the steady state values of $u, v, w, p, q, r, \phi, \theta, \psi$. This is a highly nonlinear system of equations, and it may exhibit solution branching and/or multiple solutions McKinley [Ref. 14]. Here $\dot{\psi}$ is not zero at steady state because taking $\dot{\psi}=0$ at steady state will restrict the analysis to the vertical plane. In this analysis, motion in all six degrees of freedom is studied.

Equation system (34) can be written in the following form

$$f(\bar{\mathbf{x}}, \lambda) = \mathbf{0} \quad (35)$$

Here, λ denotes any one of the system parameters. By system parameter, we mean the effects of control surfaces, propeller rpm the values of buoyancy and weight, etc. Therefore, our system of Equations (35) represent a multiparameter problem. This multiparameter problem can be solved by keeping all parameters fixed except one (denoted by λ). This system of equations for various values of λ can be solved

continuation method. The continuation method will be explained in the following section.

D. PRINCIPLES OF CONTINUATION

The system of nonlinear 'algebraic' equations

$$f (y , \lambda) = 0 \quad (36)$$

serves as a basis for the discussion. Here y denotes an n -dimensional vector. No generality is lost by restricting attention to Equation (36). Both steady state solutions of ODE's and PDE's are solved by approximating them by such system of nonlinear equations.

Assume that at least one solution of Equation (36) has been calculated. Let us denote this first solution by (y^1, λ_1) . The continuation problem is to calculate further solutions,

$$(y^2, \lambda_2), (y^3, \lambda_3), \dots \quad (37)$$

until one reaches a target point, say $\lambda = \lambda_b$. (The superscripts are not to be confused with exponents.)

The j -th continuation step starts from (an approximation of) a solution (y^j, λ_j) of equation (36) and attempts to calculate the solution (y^{j+1}, λ_{j+1}) for the next λ , namely λ_{j+1} .

With predictor-corrector methods, the step $j \rightarrow j+1$ is split into two steps:

$$\begin{array}{ccc}
 (y^j, \lambda_j) & \Rightarrow & (\bar{y}^{j+1}, \bar{\lambda}_{j+1}) \Rightarrow (y^{j+1}, \lambda_{j+1}) \\
 \text{Predictor step} & & \text{Corrector step}
 \end{array}$$

see Figure 3 Seydel [Ref. 12].

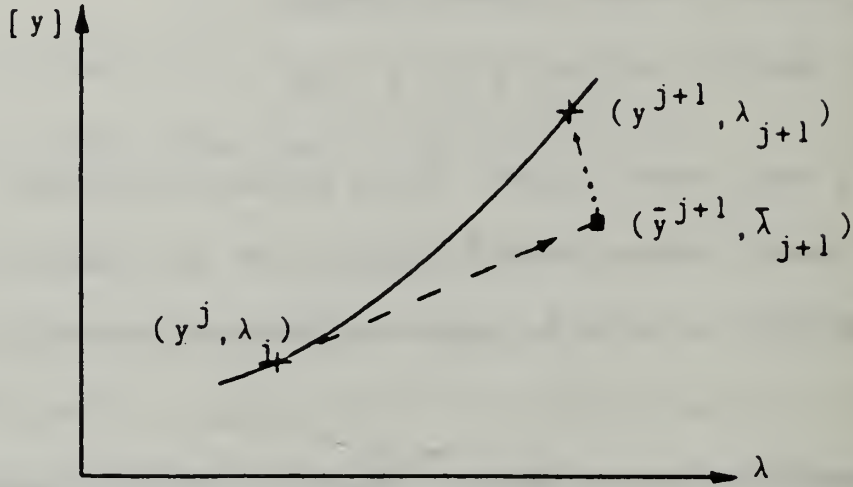


Figure 5. Schematic Showing Predictor Corrector Method (From Seydel, 1988)

In this, thesis a predictor corrector method is used. In general, the predictor is not a solution of equation (36). The predictor merely provides an initial guess for corrector iterations that home in on a solution of equation (36). The major portion of the work is either on the predictor step (resulting in an approximation close to the branch) or on the corrector step (if the predictor has produced a guess far from the branch). The distance between the two consecutive solutions (y^j, λ_j) and (y^{j+1}, λ_{j+1}) is called the step length or step size Seydel [Ref. 12].

It is obvious that first of all, we need a solution procedure for the particular class of problems under investigation. In this work, a Newton-like method has been used to find the solutions of the problem. Therefore, calculation of the first solution is often the

most difficult part. Initial guesses may be based on solutions to simplified versions of the underlying equations. Sometimes choosing special parameters (often equal to zero) leads to equations that are easily solved. In complicated cases, a hierarchy of simplifications must be gradually relaxed step by step, the solution of each equation serving as an initial guess to the following more complicated equation. Solving a sequence of equations with diminishing degree of simplification until finally the full equation solved is called homotopy Seydel [Ref. 12].

Equation system (35) can be simplified by assuming the angle of yaw (ψ), its derivative, and p , q , r to be zero. By doing this, the motion of the vehicle will be restricted to the vertical plane. In this case, the resulting equations can be solved analytically. These analytical solutions were done in McKinley [Ref. 14]. In this work these solutions and certain numerical simulation results have been used as initial conditions to the continuation runs.

The continuation approach does not guarantee that all possible solutions can be located in a specific example. In particular, changes to detect isolated branches are small. Therefore, in this work there might be other solutions which had not been located by the continuation algorithm which is used. In this thesis, a continuation method is implemented BIFPACK [Ref. 15]. A careful combination of analytical techniques, continuation methods, and numerical simulations are used to ensure that all steady state solutions are captured.

III. CONTINUATION IN δs

A. GENERAL

In this chapter, the results of continuation in terms of the sternplane angle will be discussed. As was mentioned before, it is very important to have an initial condition before starting a continuation study. In his master's thesis, McKinley solved the system of equations in the vertical plane [Ref. 14]. In that work, he found that when the longitudinal center of buoyancy (X_{GB}) was equal to -1 percent of the vehicle length, for a certain value of sternplane, deflection the real part of one of the system eigenvalues became zero while the others stayed negative. By using this fact, it was assumed that there is a pitchfork bifurcation in our system for the above specific values of system parameters. In this work, the results of his thesis were used as initial conditions in the continuation runs to find the vertical plane solutions. Since these results were the analytical solutions of the simplified system equations by restricting the motion in vertical plane, we were able in this work to validate the continuation runs. In continuation runs, the system of equations Equation (35) was solved for changing values of the primary parameter λ (in this case $\lambda = \delta s$). After a few continuation runs, a pitchfork bifurcation was found. This pitchfork bifurcation is explained in the following section.

B. PITCHFORK BIFURCATION

Suppose we have the nonlinear system of state equations,

$$\dot{x} = f(x) \quad (38)$$

we know that the equilibrium points, $\bar{x}$ of the system are defined by:

$$f(\bar{x}) = 0 \quad (39)$$

This is a nonlinear system of algebraic equations, and it may have multiple solutions in $\bar{x}$, which means that the nonlinear system may have more than one position of static equilibrium. If we pick one equilibrium $\bar{x}$, we can establish its stability properties by linearization. The linearized system becomes

$$\dot{x} = Ax \quad ,$$

where A is the Jacobian Matrix of $f(x)$ evaluated at $\bar{x}$,

$$A = \left. \frac{\partial f}{\partial x} \right|_{\bar{x}} \quad ,$$

and the state x has been redefined to designate small deviations from the equilibrium $\bar{x}$,

$$x \rightarrow x - \bar{x}.$$

As long as all eigenvalues of A have negative real parts, we know that the linear system will be stable. This means that the equilibrium $\bar{x}$ will be stable for the nonlinear system as well.

The question we ask ourselves next is, what happens if one real eigenvalue of the linearized matrix A is zero? The interesting case here is when the rest of the eigenvalues

have all negative real parts, otherwise $\bar{x}$ is unstable and the problem is solved. If the case of a zero eigenvalue appears to be too specialized to be of any practical use, consider this: Assume that $f(x)$ depends on one physical parameter, and that physical parameter is allowed to vary over some range. Then it is clear that A will depend on that parameter and as the parameter varies, it is possible that one real eigenvalue of A will become zero for a specific value of the parameter. Our problem is then to establish the dynamics of the nonlinear system as one real eigenvalue of A crosses zero; i.e., goes from negative to positive. As the solutions evolve with time, things are interesting only along the direction of the eigenvector that corresponds to the critical eigenvalue (the one that crosses zero). Along the rest of the directions in the state space, everything should converge back to equilibrium; remember that we assumed that all remaining eigenvalues of A have negative real parts.

We can see then that it is possible to approximate our original system by a one dimensional system, which is much easier to analyze. The dynamics of the two systems will be qualitatively similar. The formalization of the above reduction procedure constitutes what is known as center manifold reduction, or normal form computation in nonlinear analysis. So, let us see what happens for the case of a zero eigenvalue by using a (typical) first order system,

$$\dot{x} = \lambda x - x^3 \quad ,$$

for $\lambda > 0$ there are two nontrivial equilibria, $x = \pm \sqrt{\lambda}$. The transition of stability is illustrated by Figure 6. We can summarize the stability as follows:

- For $\lambda < 0$ only the trivial equilibrium exists and is stable.
- For $\lambda > 0$ the trivial equilibrium becomes unstable and a pair of symmetric stable equilibria are generated.

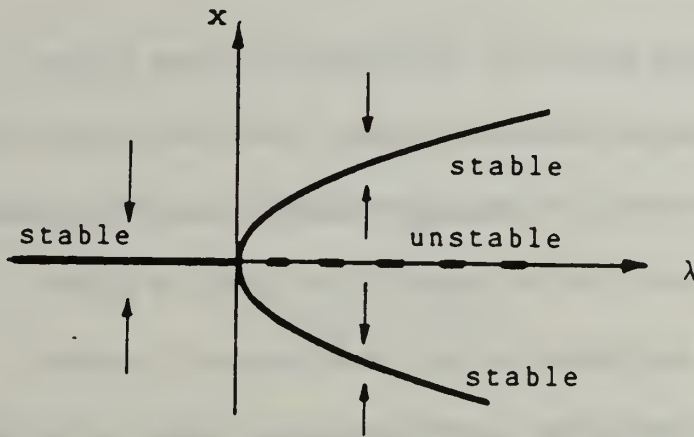


Figure 6. Supercritical Pitchfork Bifurcation

This phenomenon, the loss of stability of an equilibrium and the generation of additional equilibrium states, is called a pitchfork bifurcation and is very common in nature; Euler buckling of a beam is a very typical example. In particular, the above case is referred as the supercritical pitchfork, this is a rather benign loss of stability since upon loss of stability of the trivial equilibrium the additional nearby equilibrium states are stable. Occasionally, the above case is referred to as a soft loss of stability, since for small values of λ beyond its critical value, the final steady state of the system does not differ much from the nominal (trivial) steady state.

As a second example, consider a "similar" system as before, the linear part remains the same, and the nonlinear part x^3 changes sign,

$$\dot{x} = \lambda x - x^3$$

equilibria and stability behavior for this equation are shown in Figure 7. There is again a loss of stability at the bifurcation point $(y, \lambda) = (0, 0)$, but in contrast to the first example of Figure 6 there is no exchange of stability. Instead, the stability is lost locally at the bifurcation point. We can summarize the stability as follows :

- For $\lambda > 0$ only the trivial equilibrium exists and is unstable.
- For $\lambda < 0$ the trivial equilibrium becomes stable, and a pair of symmetric unstable equilibria are generated.

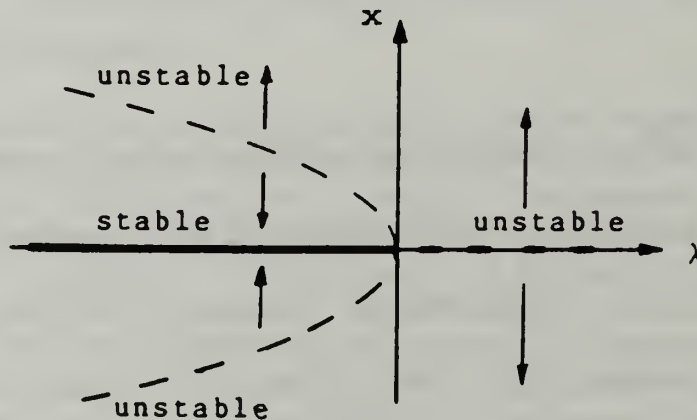


Figure 7. Subcritical Pitchfork Bifurcation

This case, which is also shown in the figure, is called a subcritical pitchfork. A comparison with the first case reveals that this is a much more serious loss of stability

case. Upon loss of stability of the trivial equilibrium position, there is no other stable equilibrium.

Now let us perturb the second example, considering the equation

$$\lambda x + x^3 + \beta = 0 \quad (40)$$

An analysis reveals that in the perturbed case the bifurcation is destroyed.

Differentiating equation (40) with respect to λ gives:

$$x + \lambda \frac{dx}{d\lambda} + 3x^2 \frac{dx}{d\lambda} = 0$$

Since $x=0$ is not a solution of equation (40), there is no horizontal tangent $dx/d\lambda=0$ to the solution curve $x(\lambda)$. Checking for a vertical tangent means considering the dependence of λ on x and differentiating with respect to x . This yields

$$\frac{d\lambda}{dx} = 0 \quad \text{for} \quad \lambda = -3x^2 ,$$

which, substituted equation (40), in turn yields the loci

$$(x, \lambda) = \left[\left(\frac{\beta}{2} \right)^{\frac{1}{3}} , -3 \left(\frac{\beta}{2} \right)^{\frac{2}{3}} \right]$$

of the vertical tangents. There are three solutions of equation (40) for

$$\lambda < -3 \left(\frac{\beta}{2} \right)^{\frac{2}{3}} ,$$

and the branching diagram looks like Figure 8 (solid lines). The point with a vertical tangent is a turning point. For $\beta < 0$, the curves in Figure 8 are reflected in the λ -axis.

As $\beta \rightarrow 0$, the curves approach the pitchfork (dashed curve). For an arbitrary perturbation β , the occurrence of a turning point is typical. Bifurcation is the rare exception because it occurs only for $\beta=0$.

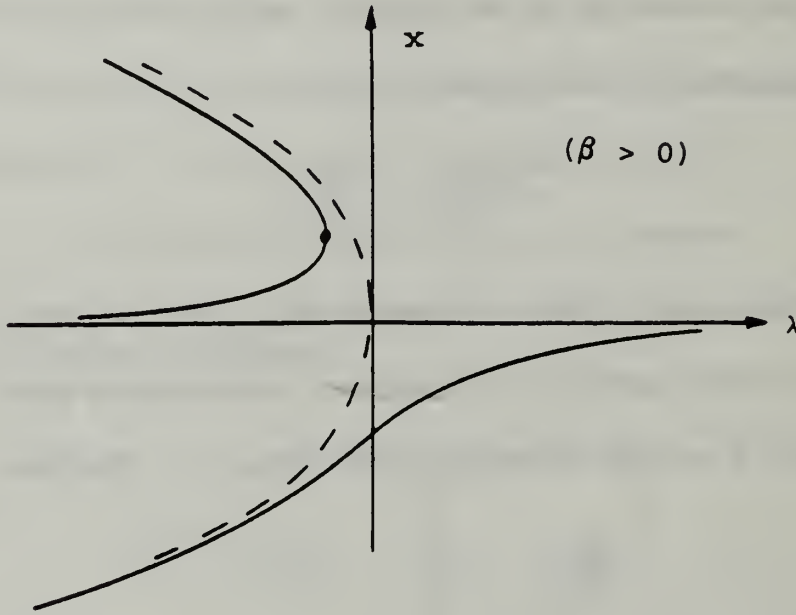


Figure 8. Perturbed Pitchfork Bifurcation

In our case, we found a pitchfork bifurcation. We found that the trivial solutions of our system come from in-plane (vertical plane) solutions. After the pitchfork bifurcation, we had a branching in the steady state solutions. The physical meaning of these branches is that they correspond to out of plane motion of the vehicle at the steady state. To get these branches, we used the numerical simulation results as an initial guess for the continuation run. Figure 9 shows a typical pitchfork bifurcation that we found is a supercritical pitchfork bifurcation. In this and all subsequent runs in this chapter, δs is

our main bifurcation parameter (λ) and δr is the perturbation parameter (β). In Figure 9 and all subsequent figures, solid lines correspond to the symmetric system ($\delta r=0$ degrees) dashed lines to $\delta r=0.5$ degrees, dotted lines to $\delta r = 2.5$ degrees, and dash-dot lines to $\delta r= 5$ degrees. As we can see in this figure, it illustrates all the typical characteristics of a pitchfork bifurcation.

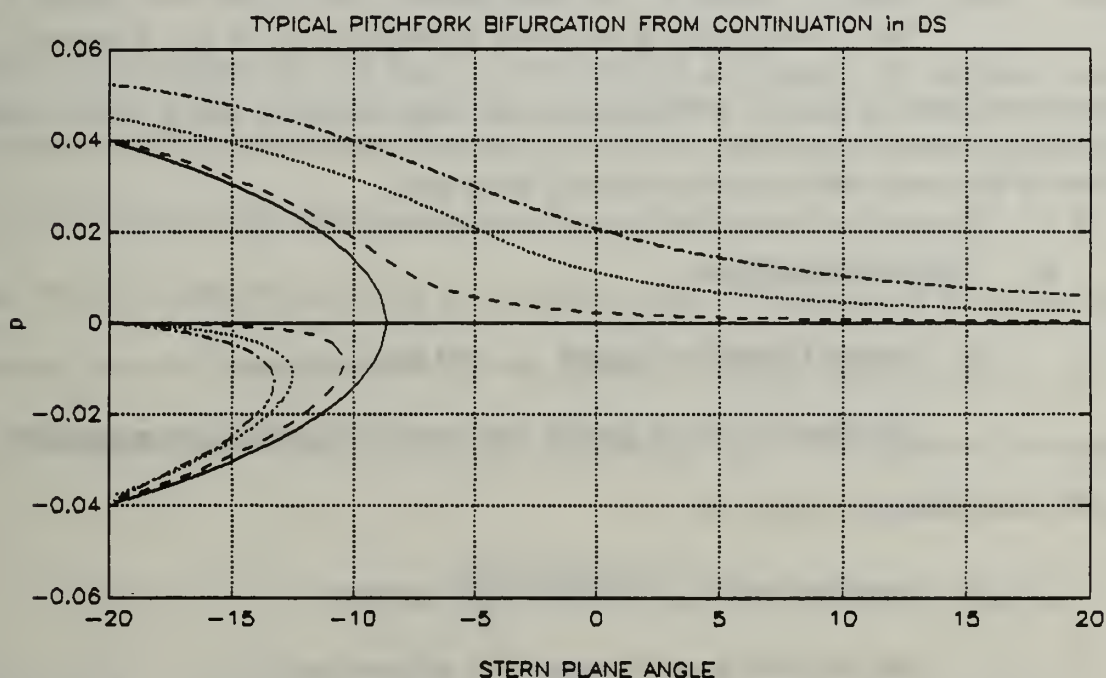


Figure 9. Pitchfork Bifurcation From Continuation Runs

C. CONDITIONS

1. Defining Additional Terms

a. *Excess Buoyancy δB*

Excess buoyancy is defined as $\delta B = B - W$ where B is the submersible's total buoyancy and W is the submersible's total weight

b. Longitudinal Center of Buoyancy , x_{GB}

The longitudinal center of buoyancy is defined as $x_{GB} = x_G - x_B$ where x_G is the longitudinal center of gravity with respect to the body fixed axis and x_B is the longitudinal center of buoyancy with respect to the body fixed axis.

c. Vertical Center of Buoyancy, z_{GB}

The vertical center of buoyancy is defined as $z_{GB} = z_G - z_B$ where z_G is the vertical center of gravity, with respect to the body fixed axis, and z_B is the vertical center of buoyancy with respect to the body fixed axis.

2. Assumed Conditions

a. Lateral Centers of Gravity, y_G , and Buoyancy, y_B

The lateral center of gravity and center of buoyancy are assumed to be on the same plane($y_G = y_B = 0$).

b. Propeller Speed , n (revolution per minute)

The propeller speed is assumed to be zero ($n=0$).

c. Propeller Coefficients , K_{prop} and N_{prop}

From Smith, Crane, and Summey [Ref. 13], the propeller coefficients are zero($K_{prop} = N_{prop} = 0$).

d. Vertical Center of Buoyancy, z_{GB}

The vertical center of buoyancy is assumed to be positive.

D. STEADY STATE RESULTS

Figures 10 through 18 show steady state solutions for the eight state variables, namely; $u, v, w, p, q, r, \Phi, \theta$. In Figures 10 through 18, solid lines correspond to $\delta r = 0$ degrees, dashed lines correspond to $\delta r = 0.5$ degree, dotted lines correspond to $\delta r = 2.5$ degrees, and dash-dot lines correspond to $\delta r = 5$ degrees. Figures 16 and 17 show steady state solutions for roll angle in two different axis scales. In all these cases, sternplane action is continuation parameter (i.e. , λ) and δr is the perturbation parameter (i.e. , β). Values of the other parameters are kept fixed: excess buoyancy $\delta B = 2\%$ of the vehicle weight (W); deflection of bow planes, $\delta b = 0$; location of horizontal and vertical centers of buoyancy, $x_B = z_B = 0$; location of vertical center of gravity, $z_{GB} = 0.1$ feet; and location of longitudinal center of buoyancy; $x_{GB} = -1\%$ of the vehicle length.

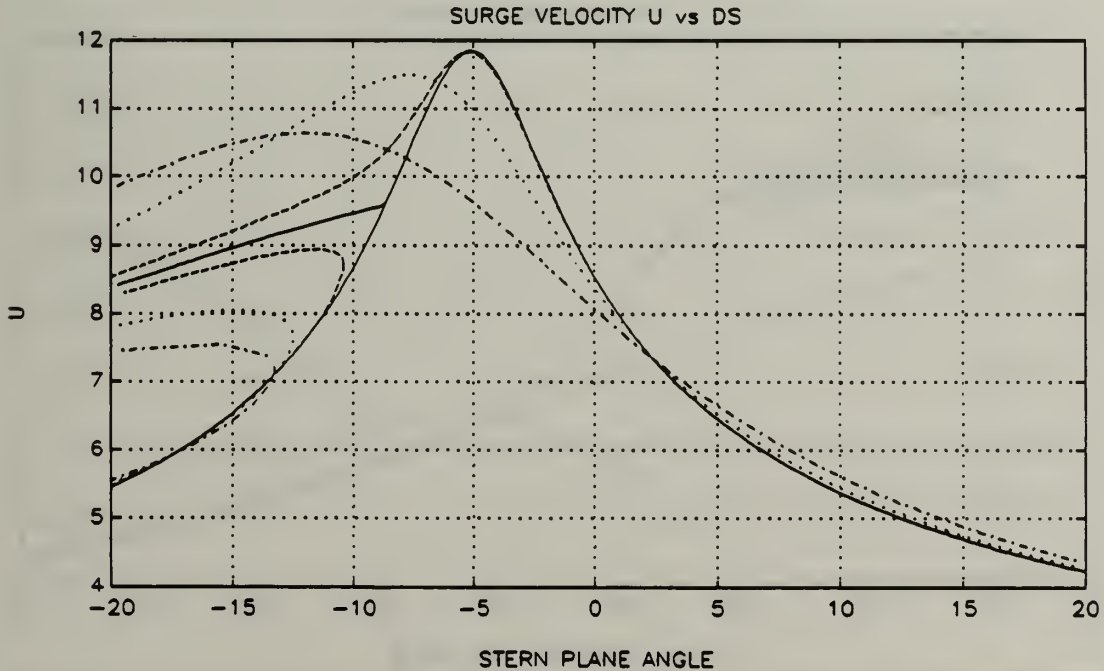


Figure 10. Steady State Surge Velocity

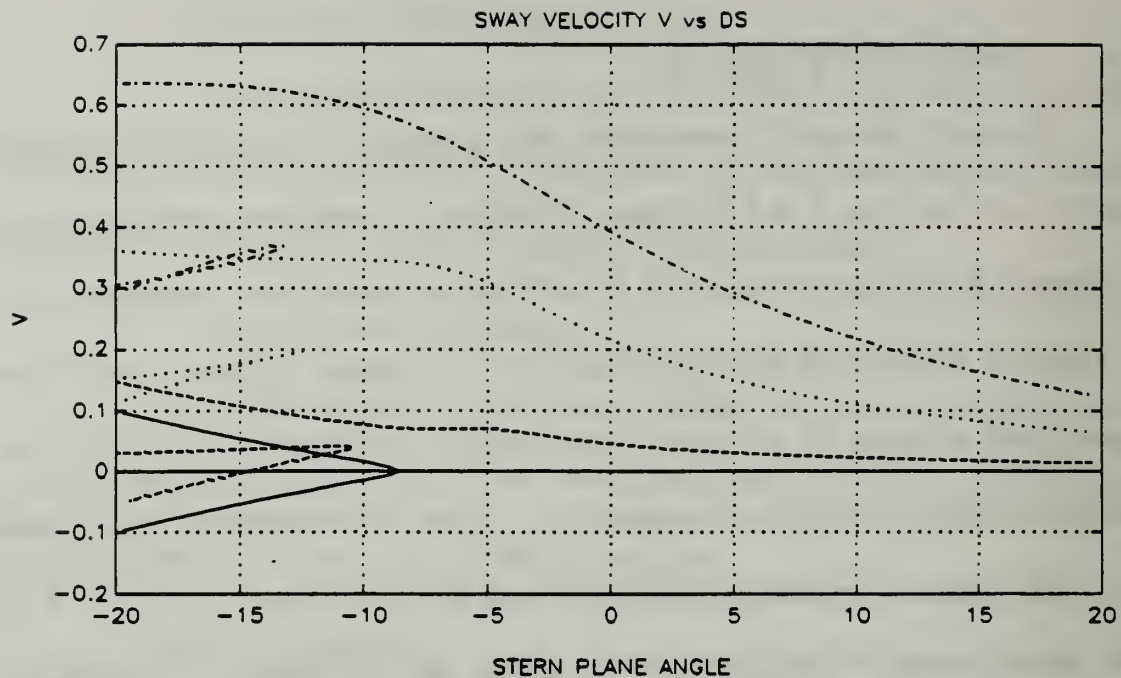


Figure 11. Steady State Sway Velocity

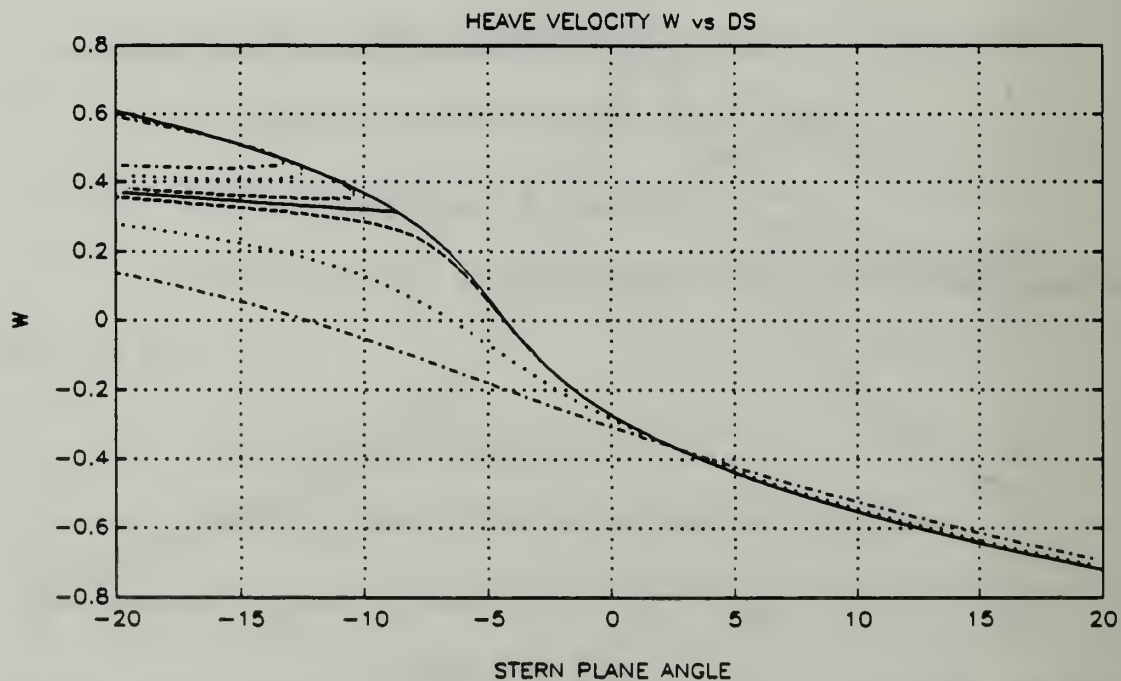


Figure 12. Steady State Heave Velocity

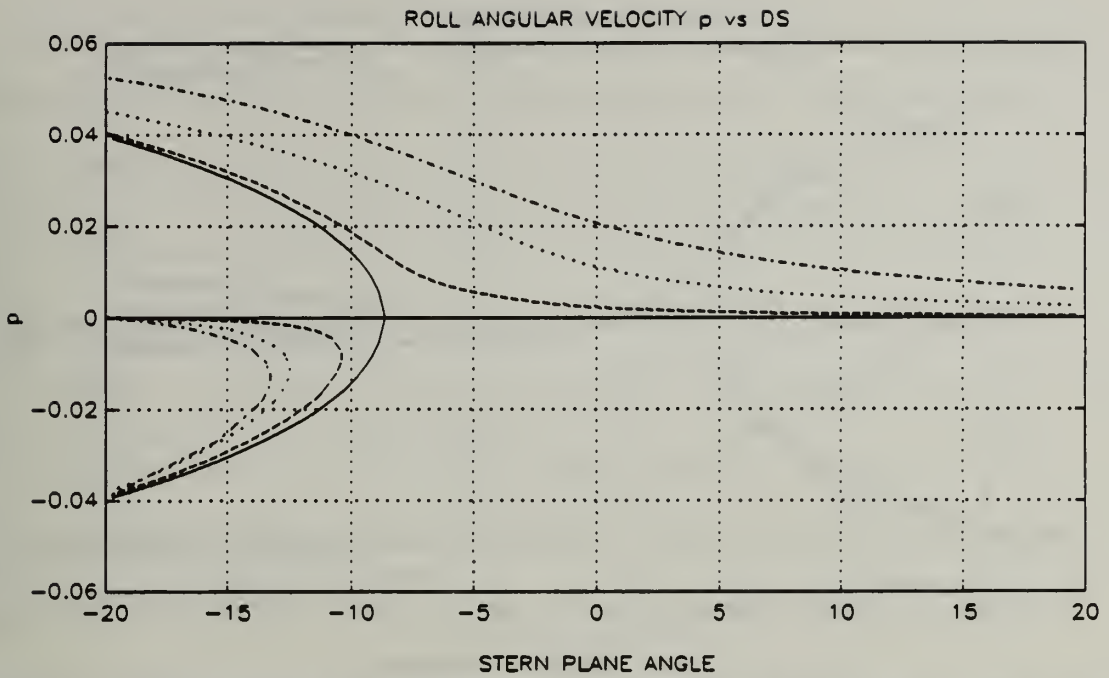


Figure 13. Steady State Roll Velocity

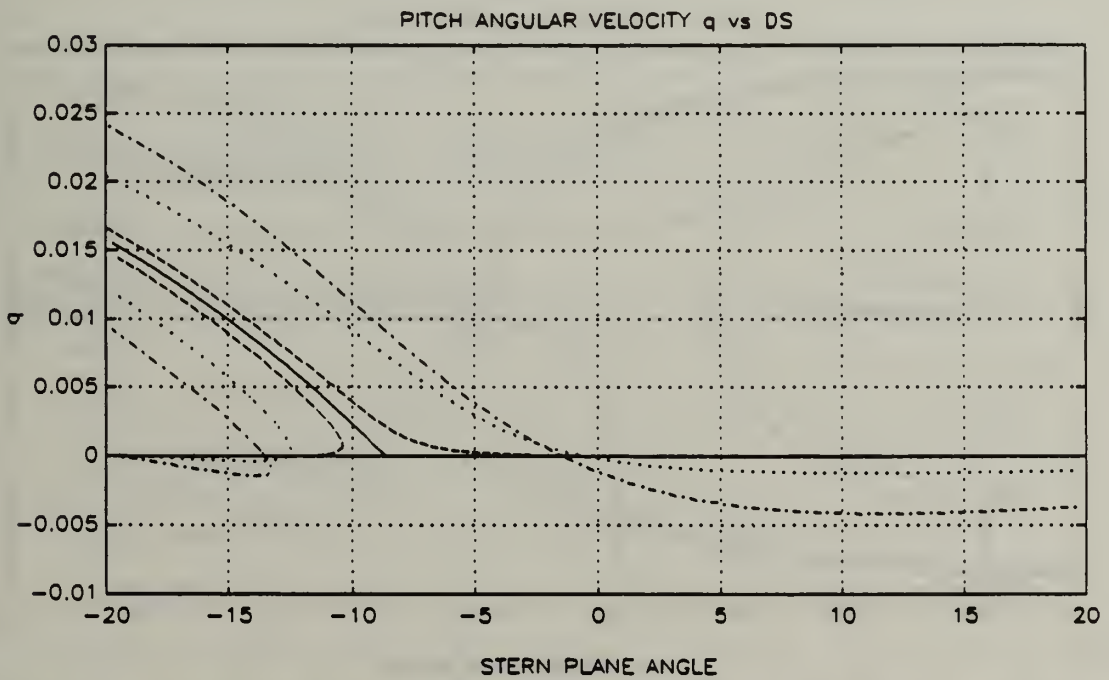


Figure 14. Steady State Pitch Angular Velocity

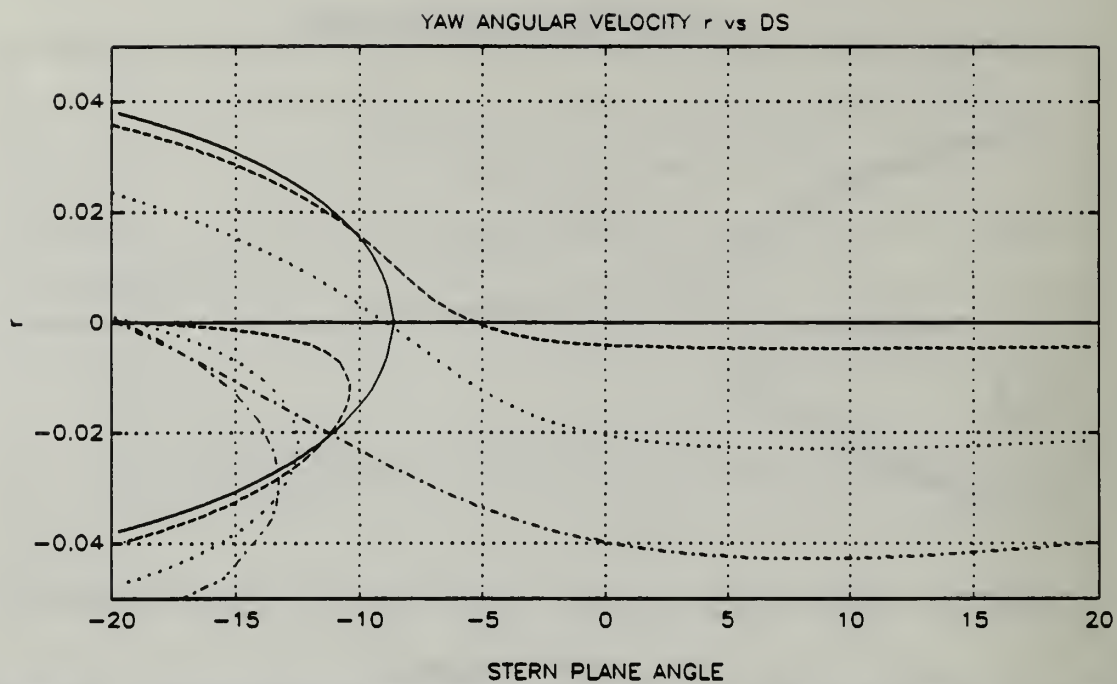


Figure 15. Steady State Yaw Angular Velocity

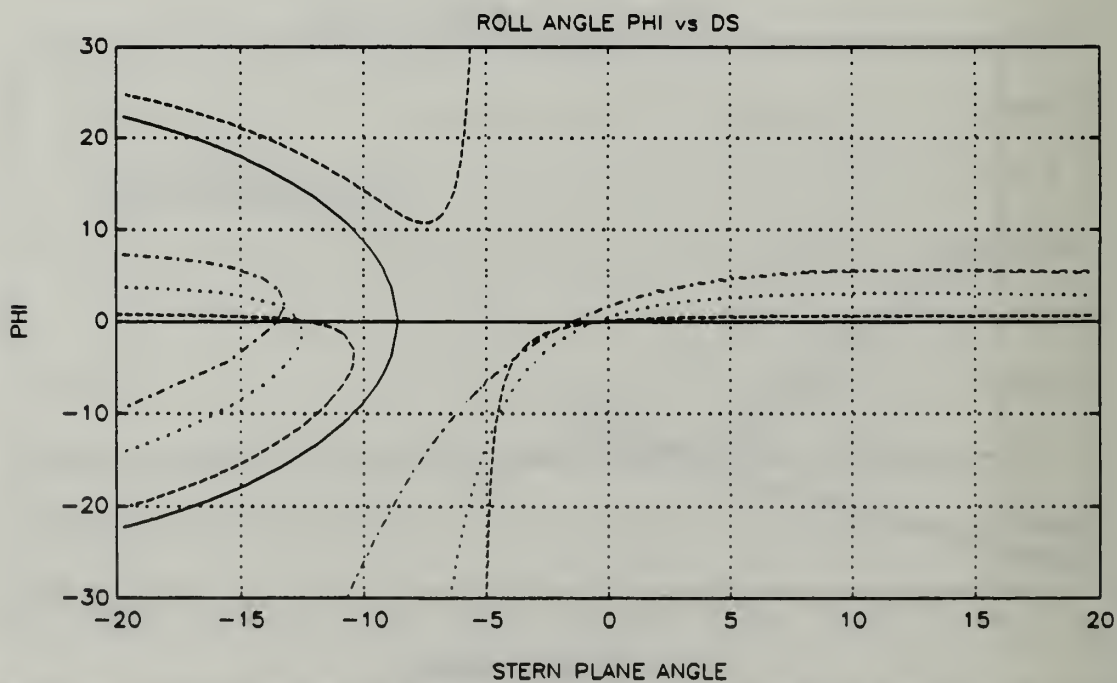


Figure 16. Steady State Roll Angle

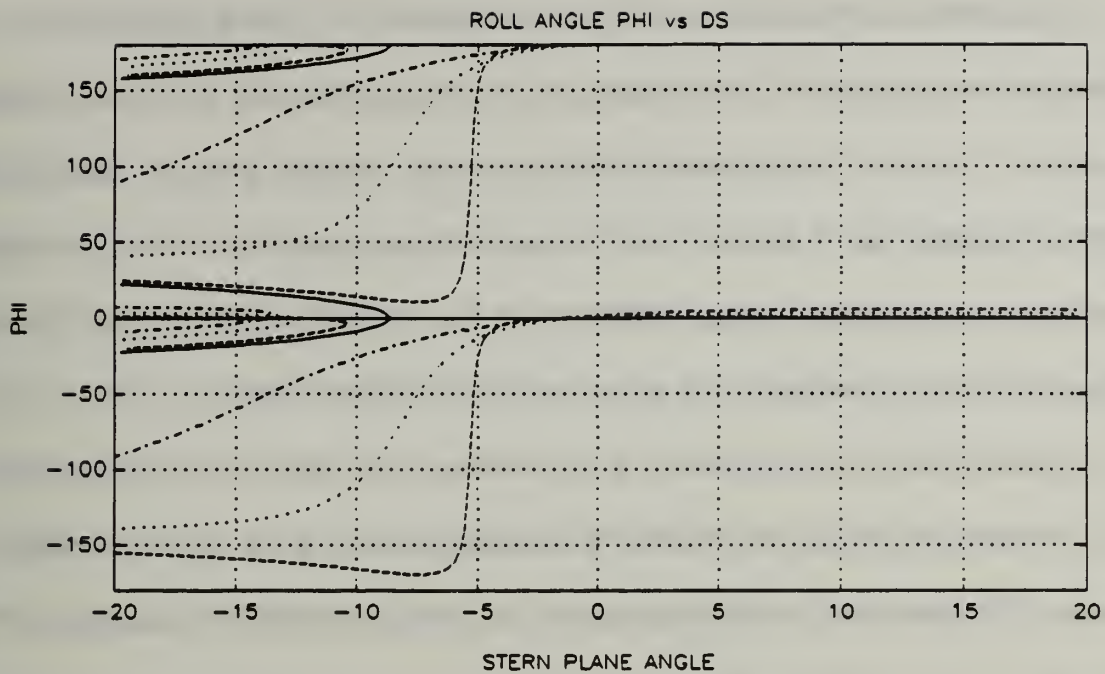


Figure 17. Steady State Roll Angle

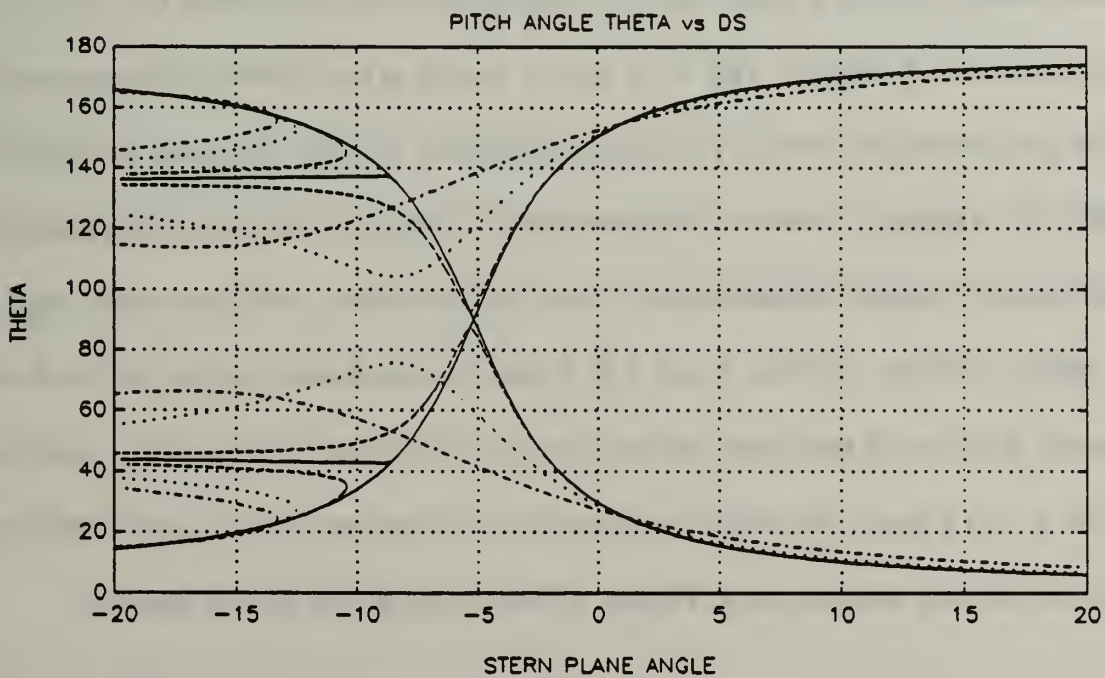


Figure 18. Steady State Pitch Angle

It can be seen that all horizontal plane variables, v , p , r and Φ , exhibit the typical characteristics of pitchfork bifurcation. For $\delta r = 0$, there exists a dive plane angle δs between -5 and -10 degrees, such that the in plane solution becomes unstable and symmetric stable out of plane solutions appear. Nonzero values of δr result, naturally, in out of plane solutions for the entire range of δs . Multiple solutions appear also at a certain value of δs , called turning point, which is a function of δr .

The vertical plane variables u , w , q , and θ are even functions of horizontal plane motions and, as a result, they exhibit "solution separation" of the in plane motions for values of δs lower than the bifurcation point. Of special interest is an examination of the steady state values of the pitch angle θ and roll angle Φ , since these are directly related to the vehicle orientation. A clear examination of the possible steady state solution, so is the pair $(\pi - \theta, \pi - \Phi)$. When $\delta r = 0$, it can be seen from Figure 18 that the stable in-plane solution exhibits a steady state pitch angle greater than 90 degrees for values of δs less than about -5 degrees. This is, of course, true up to the pitchfork bifurcation point. This phenomenon of inverted pendulum stabilization was first discussed by McKinley [Ref.14]. It appears, however, from the results of Figure 18 that the inverted pendulum stabilization is highly degenerate and is destroyed as soon as a non-zero rudder angle δr is applied. The two solutions, θ and $\pi - \theta$, do not cross each other as they did for $\delta r = 0$. Instead, they veer of each other, the lower one corresponding to $\Phi = 0$ and the upper one to $\Phi = \pi$. As a result, the vehicle configuration is always keel down, the only difference between the two solution sets at Figures 17 and 18 being 180 degrees heading.

Another important observation can be made by considering the results of Figures 16 and 17, in terms of the roll angle Φ . It appears that there exists a range of sternplane angle where the exact value of Φ is very sensitive to the exact value of δr for small values of δr . The rudder angle is related to the angle of attack of the fluid flow, which is highly dependent on such factors as vortex shedding, flow separation, and ambient flow turbulence Sarpkaya [Ref. 5]. Since these factors are to a great extent uncertain and difficult to model, situations like the one described above should be avoided in practice. This bifurcation and continuation methods used in this thesis provide a systematic and effective way of identifying precisely parameter regions where such sensitivity of the response to parameter values exists. These regions should be avoided when executing a proper recovery procedure.

IV. CONTINUATION IN δr

A. GENERAL

In this chapter, the results of continuation in terms of the rudder angle will be discussed. In this continuation run, the initial conditions were taken from the previous results of continuation in terms of the rudder angle. The configuration of the vehicle was just like the previous case. In this case the system of equations (35) was solved for changing values of the primary parameter λ (here $\lambda = \delta r$). At the end of the runs, a hysteresis behavior was observed for some of the state variables. This hysteresis behavior, which is complementary to pitchfork bifurcation, is explained in the following section.

B. HYSTERESIS PHENOMENON

When one-parameter families of equation are studied, there is no need to talk about a hysteresis phenomenon. But similarly to the pitchfork bifurcation, this situation might change when a two-parameter family of equations is studied,

$$f(y, \lambda, \alpha) = 0.$$

Now let us consider the following equation:

$$x^3 - \lambda + \alpha x = 0 \quad (41)$$

Let us call this algebraic equation as a function of x $F(x) = 0$. It can be observed that equation (41) is similar to the generic pitchfork equation (40). The only difference is that the roles of λ and α have been interchanged. The constant term is now our primary bifurcation parameter, while the coefficient of x is the perturbation parameter. We know that at the turning point F , and dF/dx , should be zero. So,

$$\frac{df}{dx} = 0 \quad \Rightarrow \quad x^2 = -\frac{\alpha}{3}$$

If we substitute this value into Equation (41), we will get the following equation.

$$27\lambda^2 + 4\alpha^3 = 0 \quad (42)$$

This equation gives the relationship between λ and α at the vertical tangent. Now we

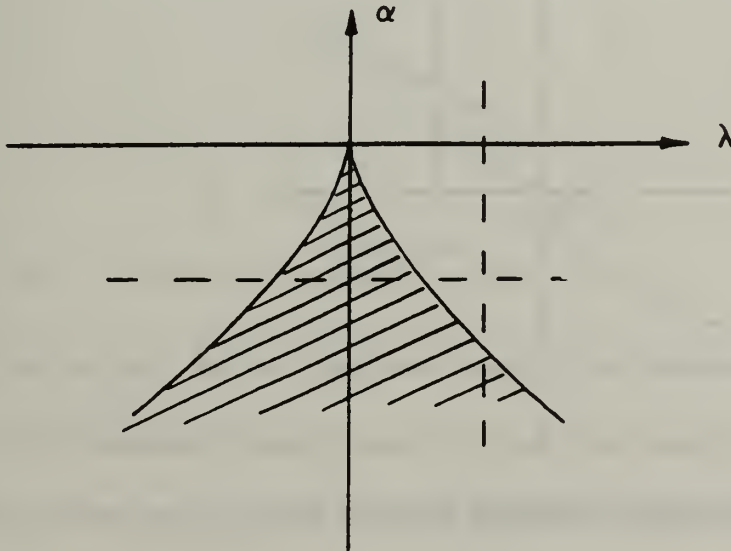


Figure 19. Cusp Curve

consider equation (41) as a two parameter model with λ and α having equal rights. Equation (42) establishes a curve in the (α, λ) plane, which takes the form of a cusp (Figure 19), (Cusp curves will be explained in a more general context in Chapter VIII.).

For all combinations of (α, λ) values inside the cusp (hatched region), Equation (41) has three solutions; for values outside the cusp, there is only one solution. This becomes clear when branching diagrams are drawn. As a generic behavior, if we draw branching diagrams for different values of α when λ is the primary continuation parameter, we get the Figures 20, 21, 22 according to the sign of α .

- $\alpha = 0$
- $\alpha > 0$
- $\alpha < 0$

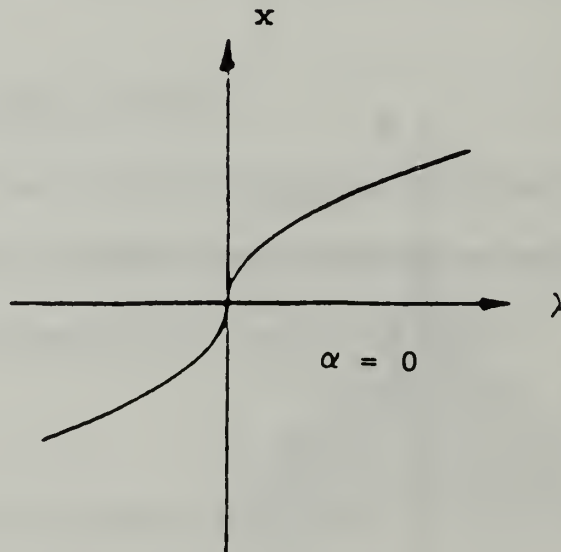


Figure 20. Branching Diagram When $\alpha = 0$

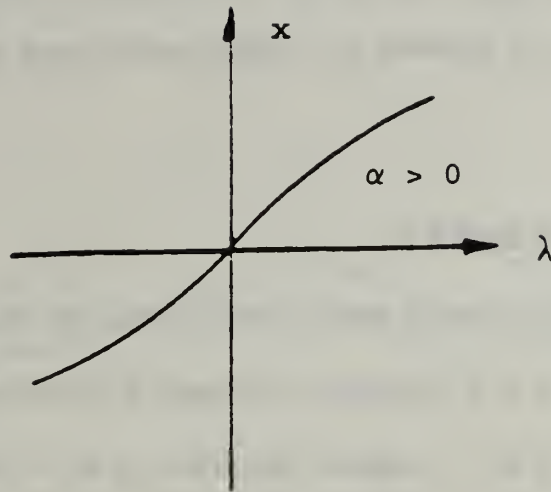


Figure 21. Branching Diagram When $\alpha < 0$

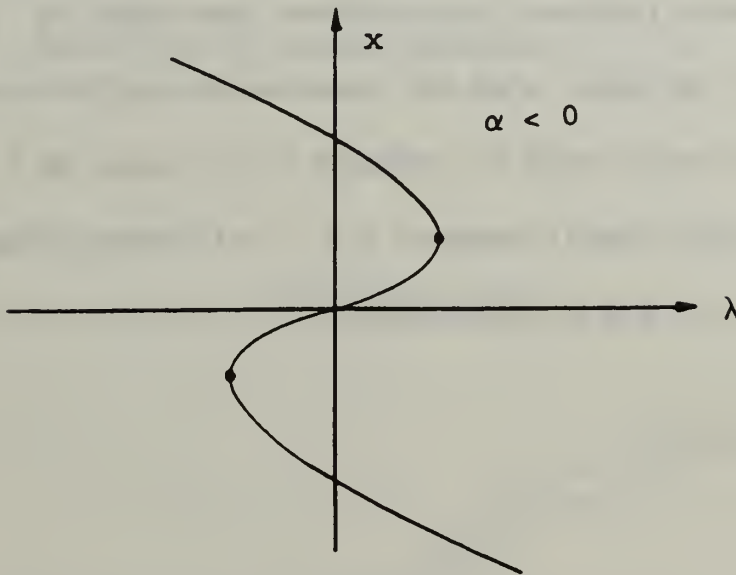


Figure 22. Branching Diagram When $\alpha > 0$

In this last case, we see a hysteresis effect. When hysteresis occurs in a branch, there are two associated turning points with hysteresis behavior. These points occur precisely at the two real solutions of λ from Equation 42. Here there is a jump when the solution is on an appropriate part of the branch. The two outer solution in x are stable

while the inner most solution between the two turning points is unstable. The two dotted straight lines of Figure 19 represent two distinct paths through the cusp and explained in Chapter VIII.

C. STEADY STATE RESULTS

Figures 23 through 30 show steady state solutions for the eight state variables, namely; u , v , w , p , q , r , Φ , θ . In Figures 23 through 30 solid lines correspond to $\delta s = 0$ degree, dashed lines to $\delta s = -5$ degrees, dotted lines to $\delta s = -10$ degrees and dash-dot lines to $\delta s = -20$ degrees. For these continuation runs, the rudder angle (δr) is the primary continuation parameter (i.e., λ) and stern plane angle (δs) is perturbation parameter (i.e., α) and values of all other parameters are kept fixed: excess buoyancy, $\delta B = 2\%$ of the vehicle weight (W); deflection of bow planes, $\delta b = 0$; location of horizontal and vertical centers of buoyancy, $x_B = z_B = 0$; location of longitudinal center of buoyancy, $x_{GB} = -1\%$ of the vehicle length (L).

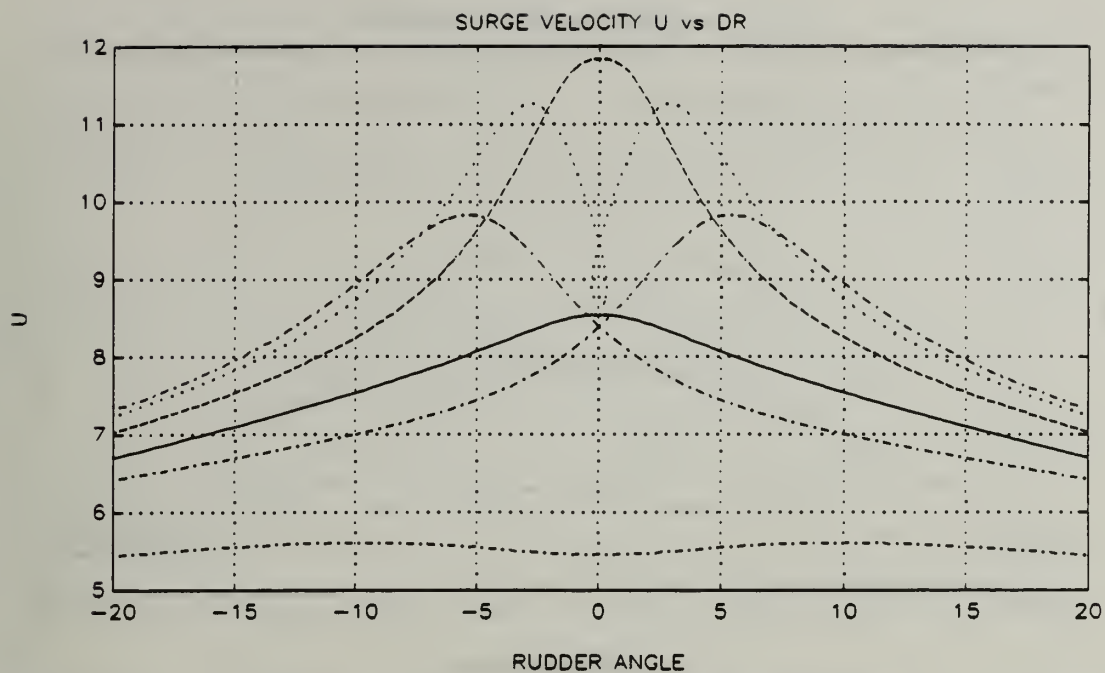


Figure 23. Steady State Surge Velocity

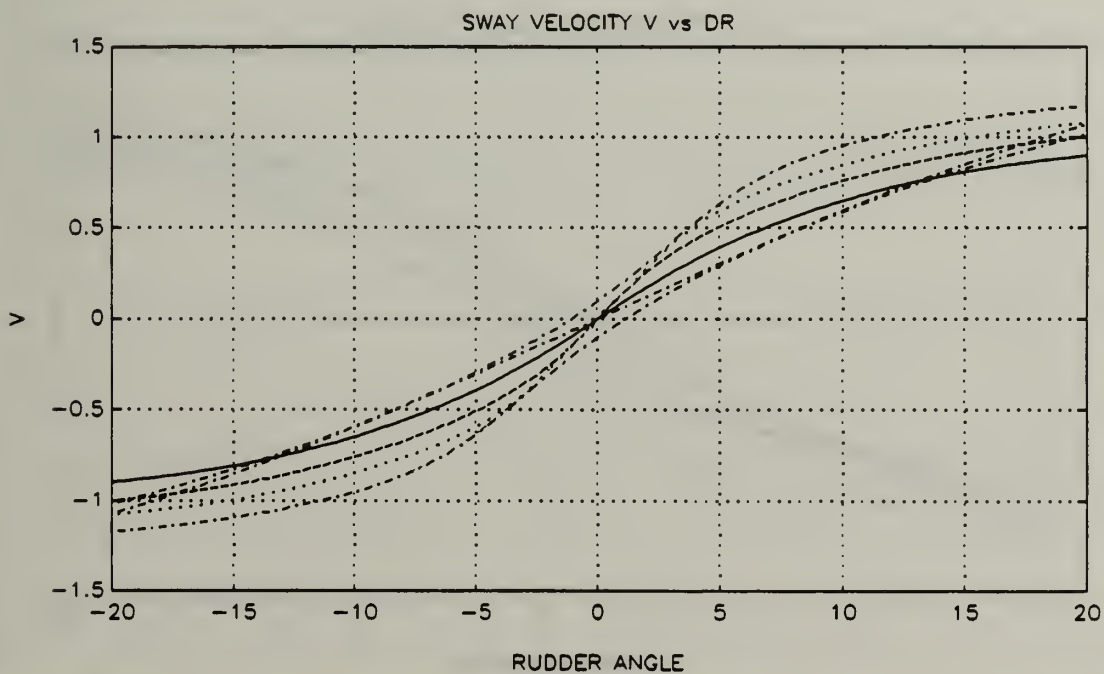


Figure 24. Steady State Sway Velocity

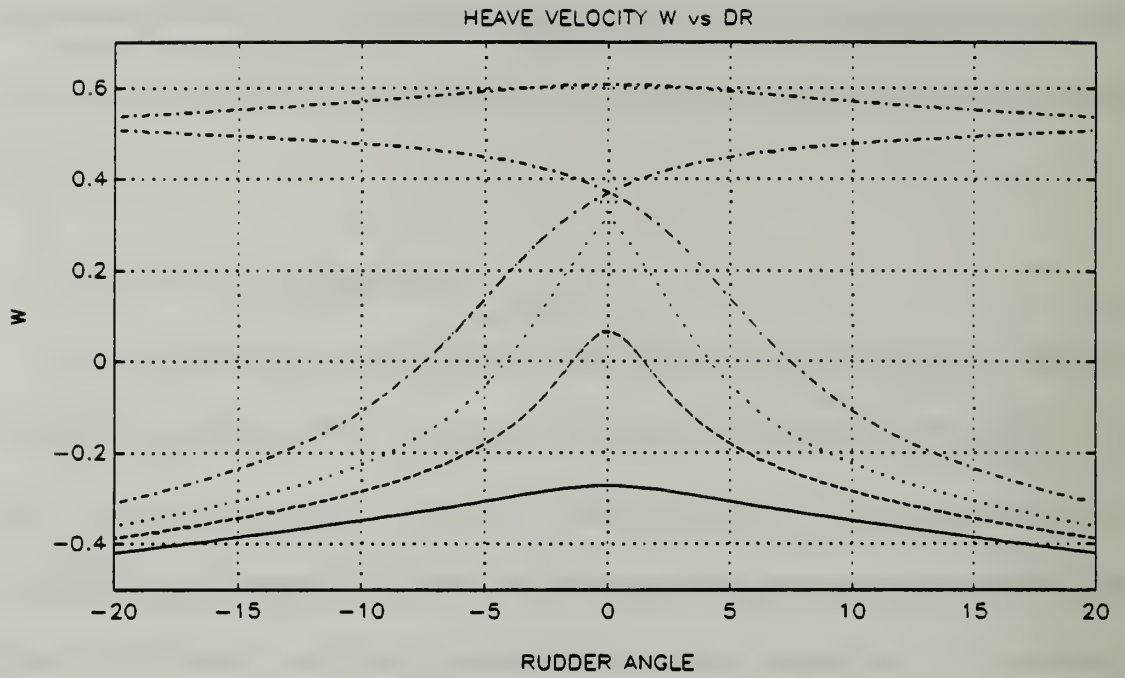


Figure 25. Steady State Heave Velocity

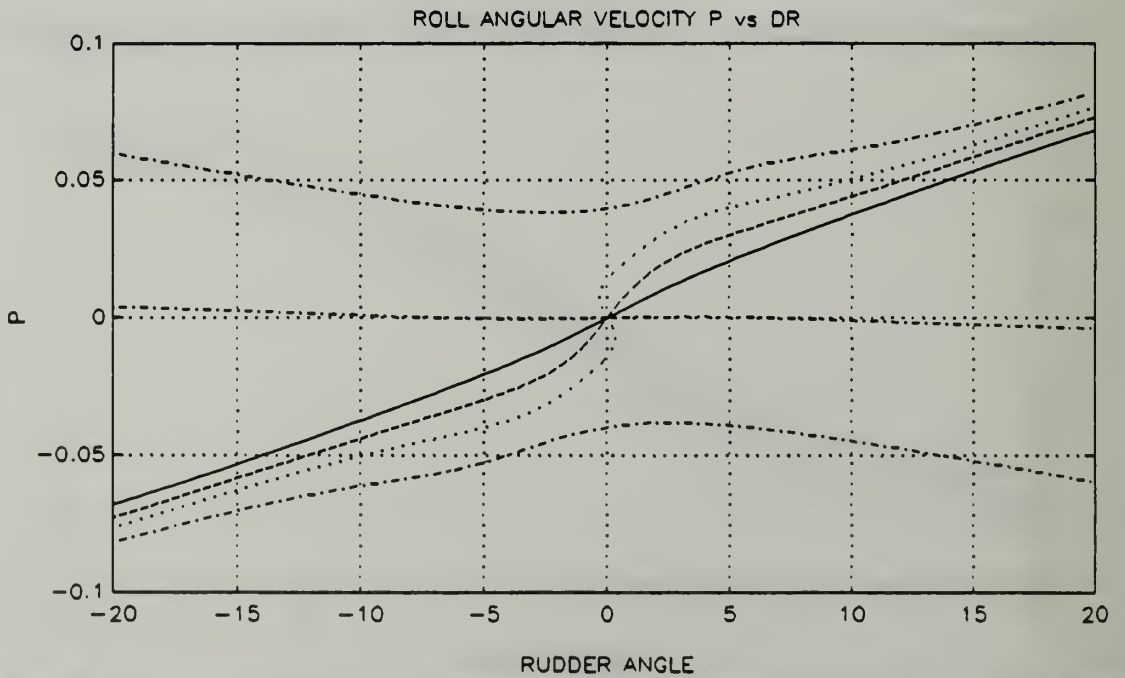


Figure 26. Steady State Roll Angular Velocity

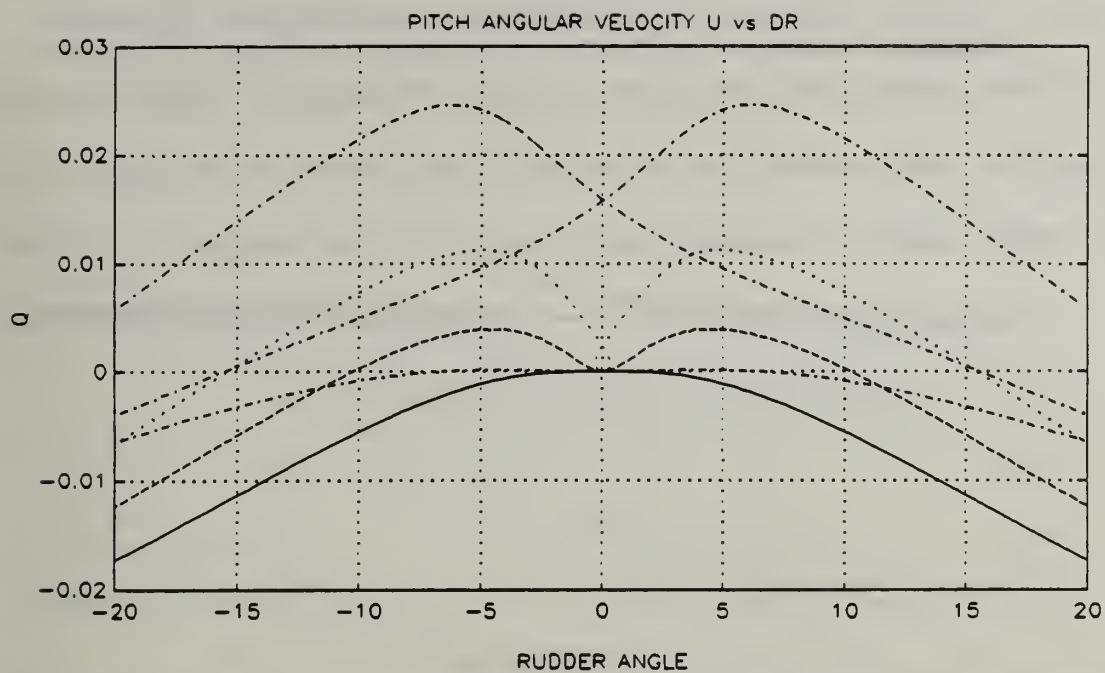


Figure 27. Steady State Pitch Angular Velocity

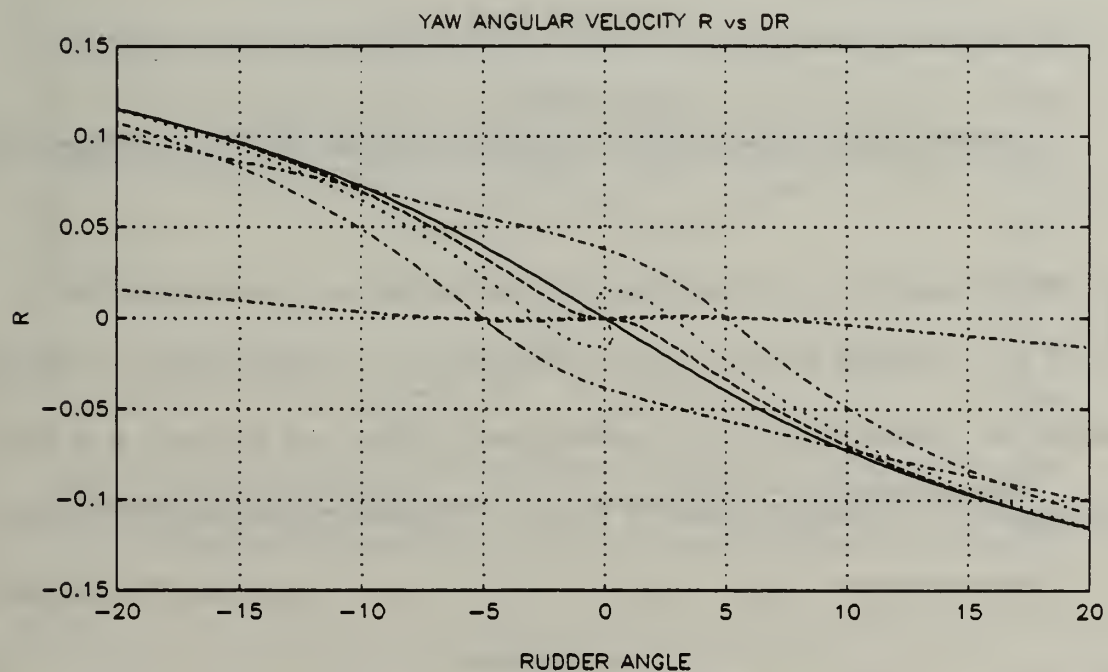


Figure 28. Steady State Yaw Angular Velocity

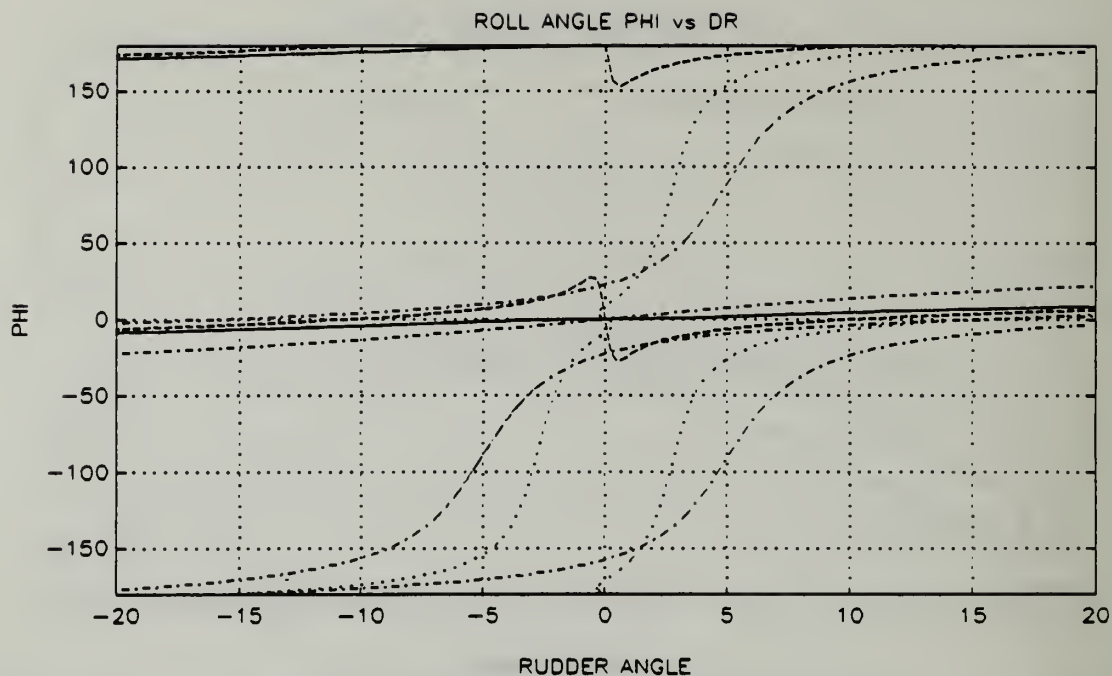


Figure 29. Steady State Roll Angle

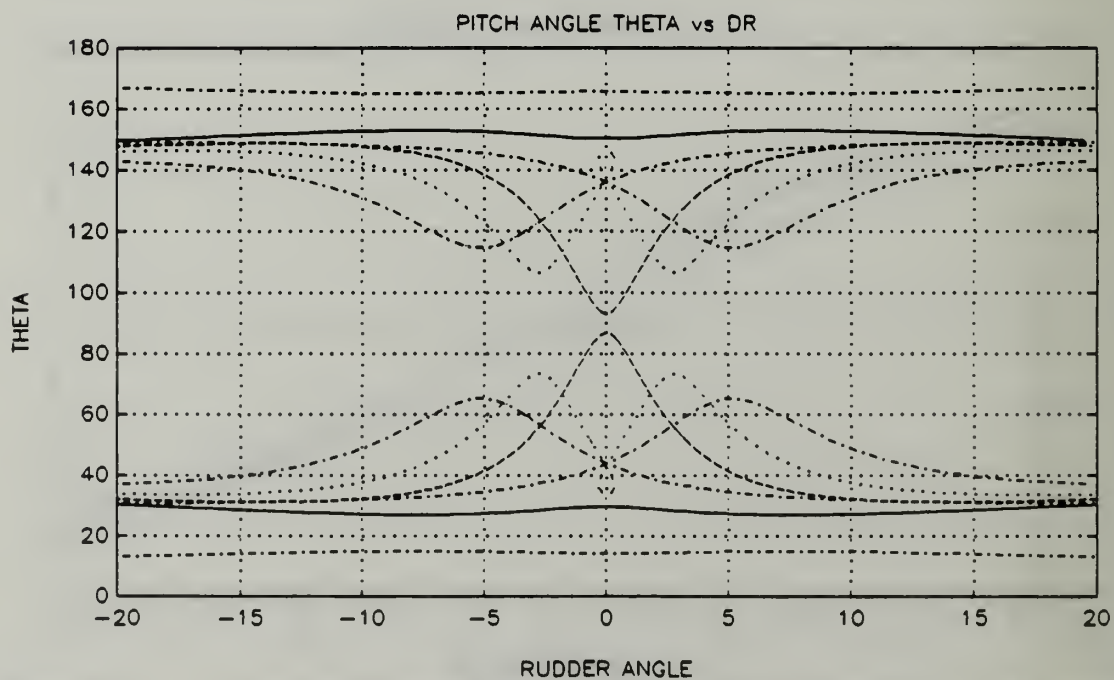


Figure 30. Steady State Pitch Angle

Figures 23 through 30 show that the horizontal plane state variables v , p , r , Φ behave like Figure 22 and exhibit a hysteresis behavior for certain values of δs . If we zoom in Figure 28, we obtain Figure 31 and can see this hysteresis effect very clearly. When $\delta s = -15$ degrees, there is a typical hysteresis phenomenon. Figures 24, 26, and 28 also show the same kind of behavior.

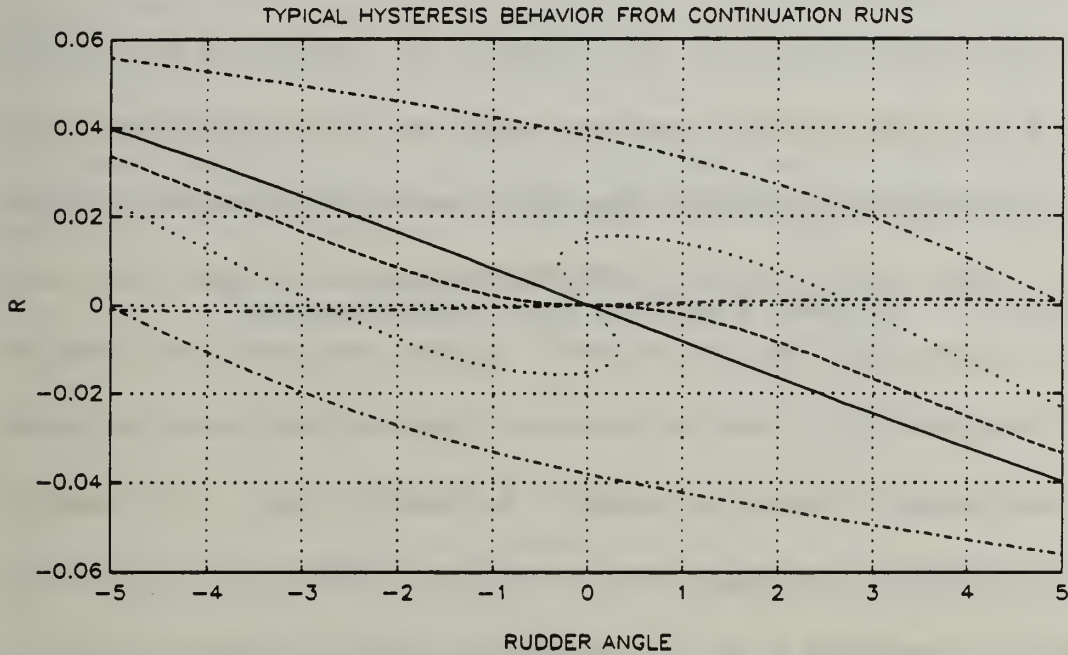


Figure 31. Typical Hysteresis Effect from Continuation

On the other hand, the vertical plane state variables u , w , q , θ behave like an even function in x (for example $|x|$). This behavior can be seen in Figures 23, 25, 27, 30. where it is clear that they exhibit generic behavior of an even function for different values of the perturbation parameter (i.e, δs). If we zoom in Figure 23, we can see this behavior, very clearly.

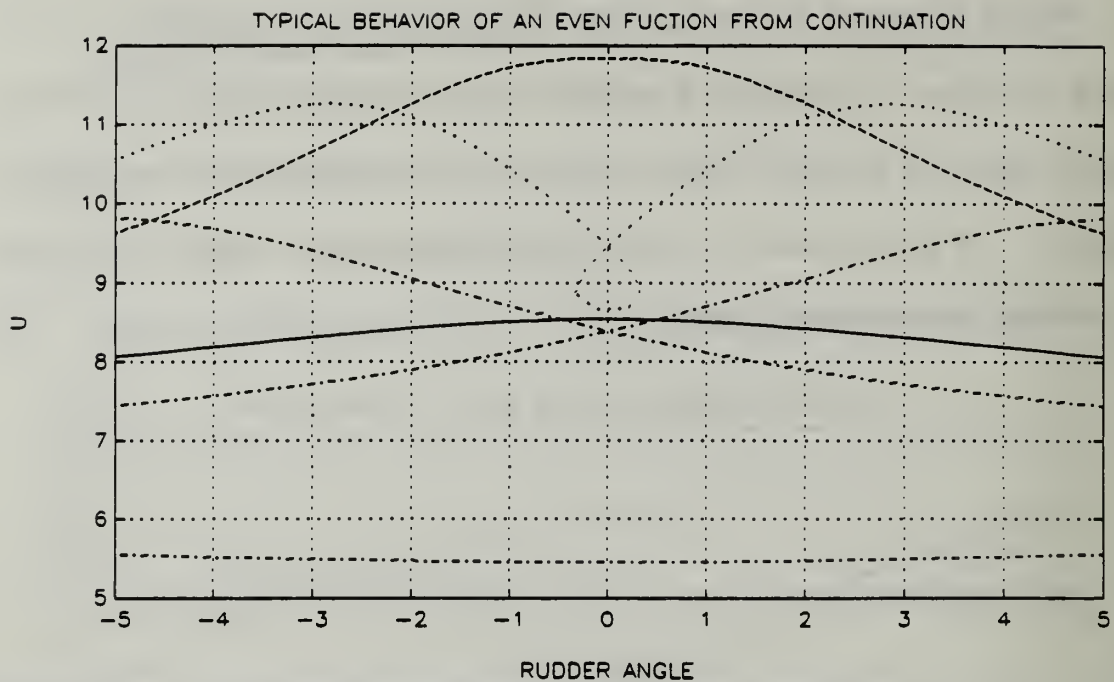


Figure 32. An Even Function from Continuation

V. CONTINUATION IN x_{GB}

A. GENERAL

In this chapter, the results of continuation in terms of the longitudinal center of buoyancy will be discussed. In continuation runs, the system of equations (35) was solved for changing values of the primary parameter λ where in this case $\lambda = x_{GB}$. In these runs, first we fixed the value of rudder angle and changed the values of stern plane angle, then we kept the sternplane angle fixed and changed the rudder angle, to see how the steady state values were changing. When we keep the rudder angle at zero and change the values of sternplane angle, we observe that there is a pitchfork bifurcation for the states v , p , r . Then, we fixed δs at -20 degrees and started to change rudder angle to see how the pitchfork bifurcation diagrams were changing with changing rudder angle. As a last continuation run for this configuration, the value of sternplane angle was fixed at zero and we changed the value of rudder angle. We didn't see any solution branching for this case.

B. STEADY STATE RESULTS

1. $\delta r = 0$ Degrees, δs is Perturbation Parameter

Figures 33 through 40 show steady state solutions for eight state variables, namely; u , v , w , p , q , r , Φ , θ . In Figures 33 through 40, solid lines correspond to $\delta s = 0$ degrees, dotted lines correspond to $\delta s = -5$ degrees, and dash-dot lines correspond to $\delta s = -$

20 degrees. In graphs of the state, variables v , p , r , θ dash-dot lines may seem like solid lines; this is because the continuation between the parameter values $x_{GB} = -13$ degrees and $x_{GB} = .7$ degrees traced the same curve twice in order to make sure that all solutions for all the state variables were captured. Therefore, out of plane solutions in this parameter range belong to sternplane angle $\delta s = -20$ degrees.

In all these cases, the longitudinal center of buoyancy is the continuation parameter and δs is the perturbation parameter, while δr is kept at zero degrees. Values of the other parameters are kept fixed: excess buoyancy, $\delta B = 2$ % of the vehicle weight (W); deflection of the bow planes, $\delta b = 0$; location of horizontal and vertical centers of gravity, $x_B = z_B = 0$; location of vertical center of gravity, $z_{GB} = .1$ feet.

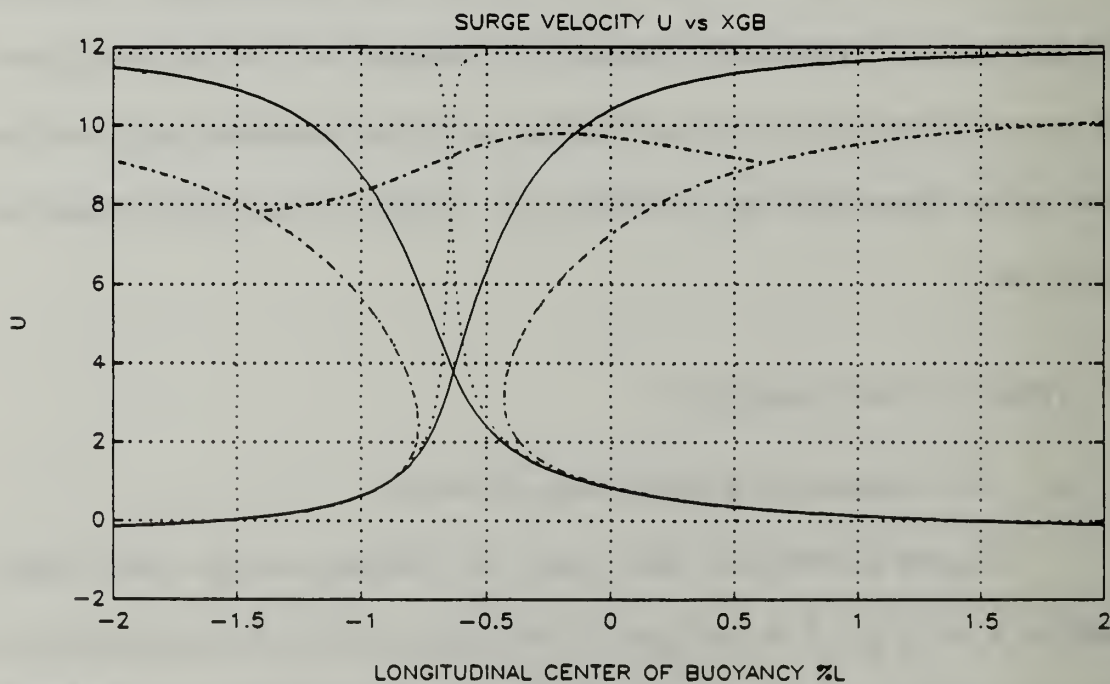


Figure 33. Steady State Surge Velocity

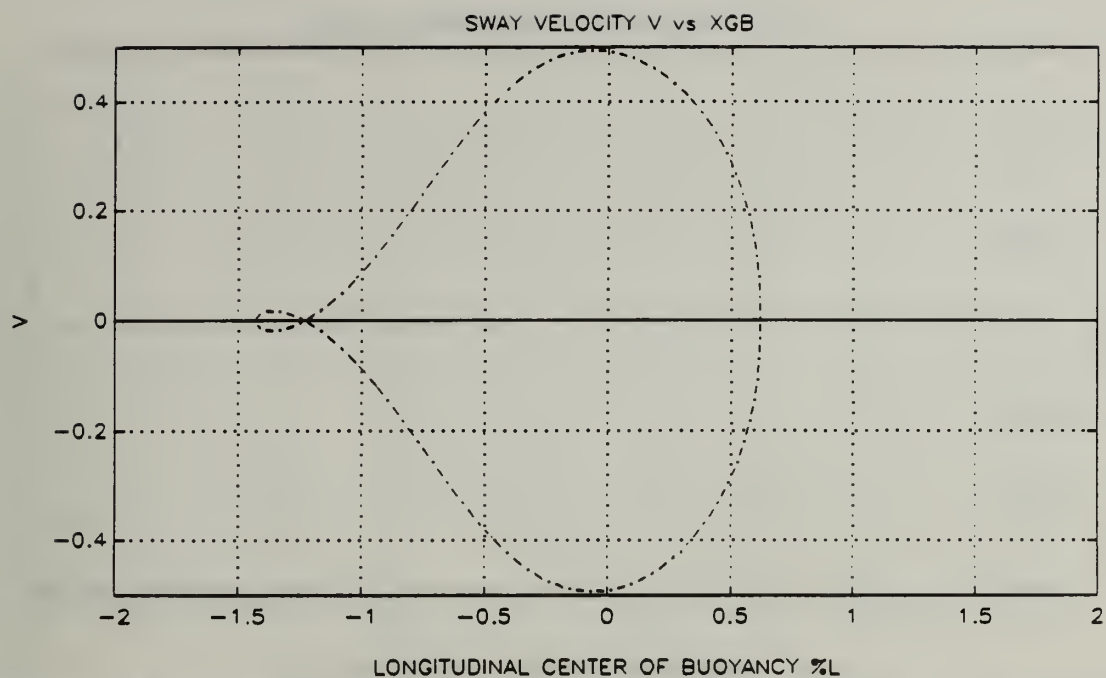


Figure 34. Steady State Sway Velocity

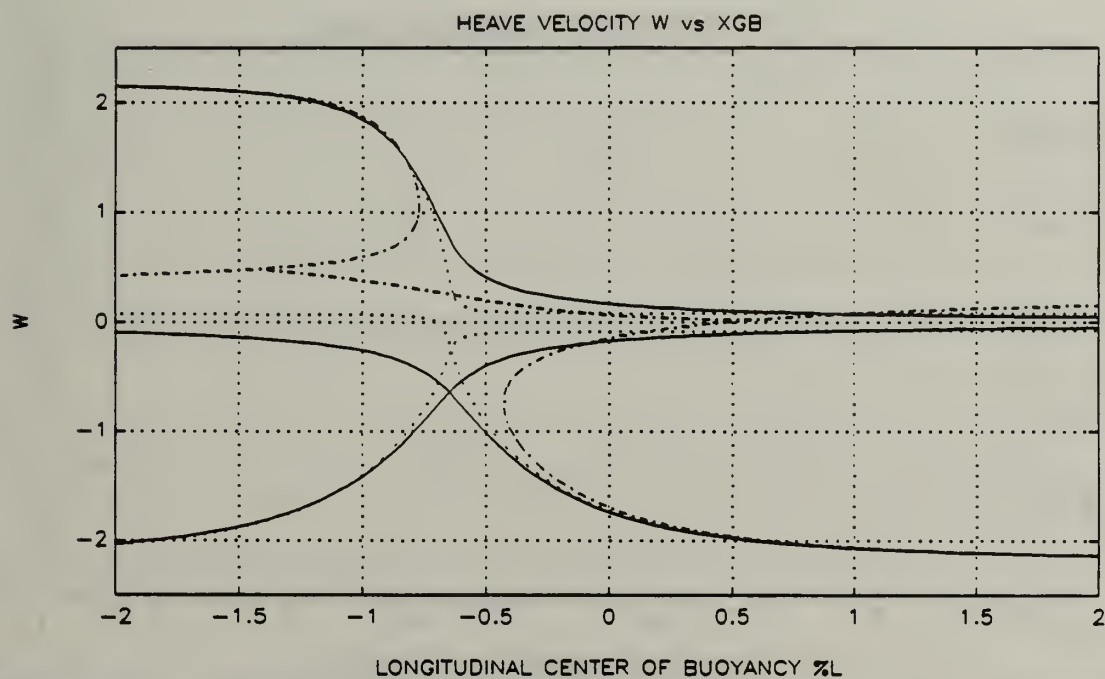


Figure 35. Steady State Heave Velocity

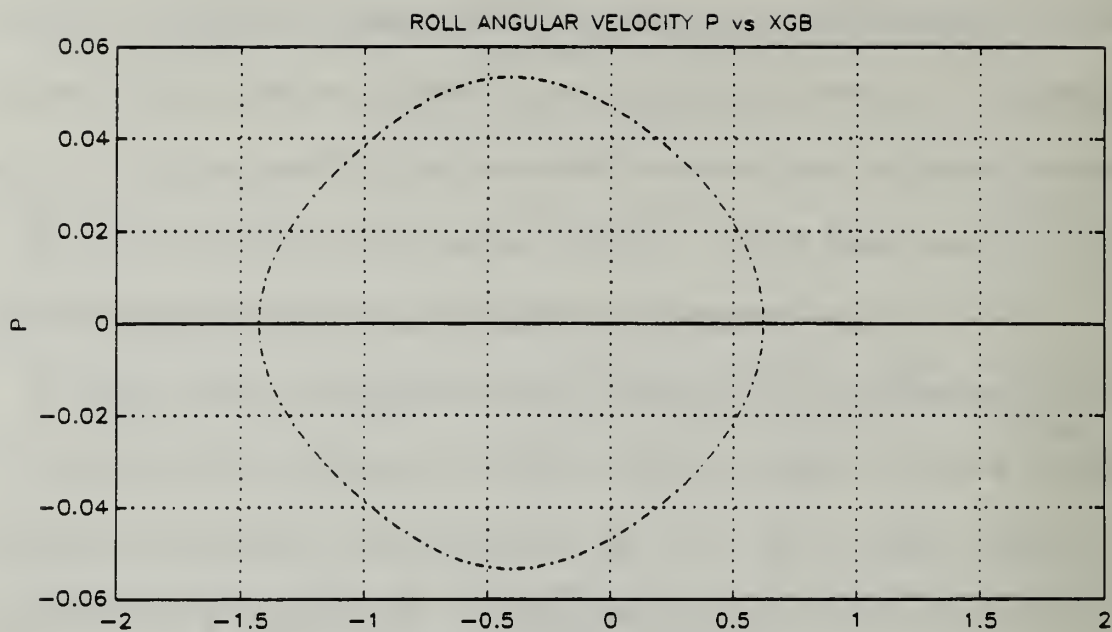


Figure 36. Steady State Roll Angular Velocity

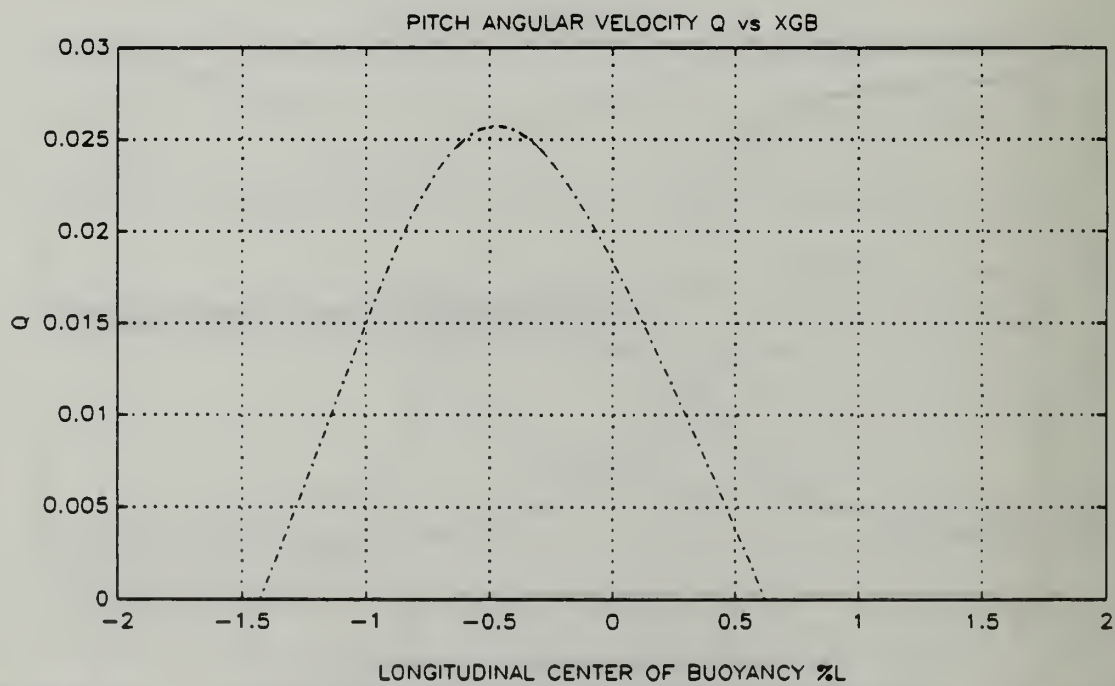


Figure 37. Steady State Pitch Angular Velocity

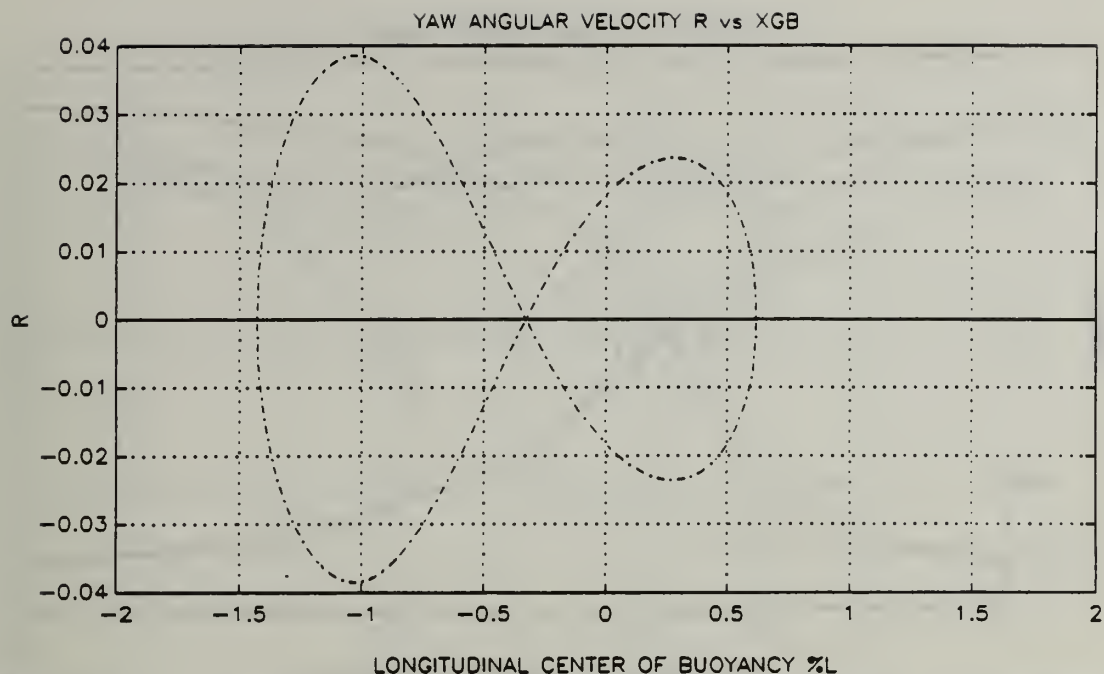


Figure 38. Steady State Yaw Angular Velocity

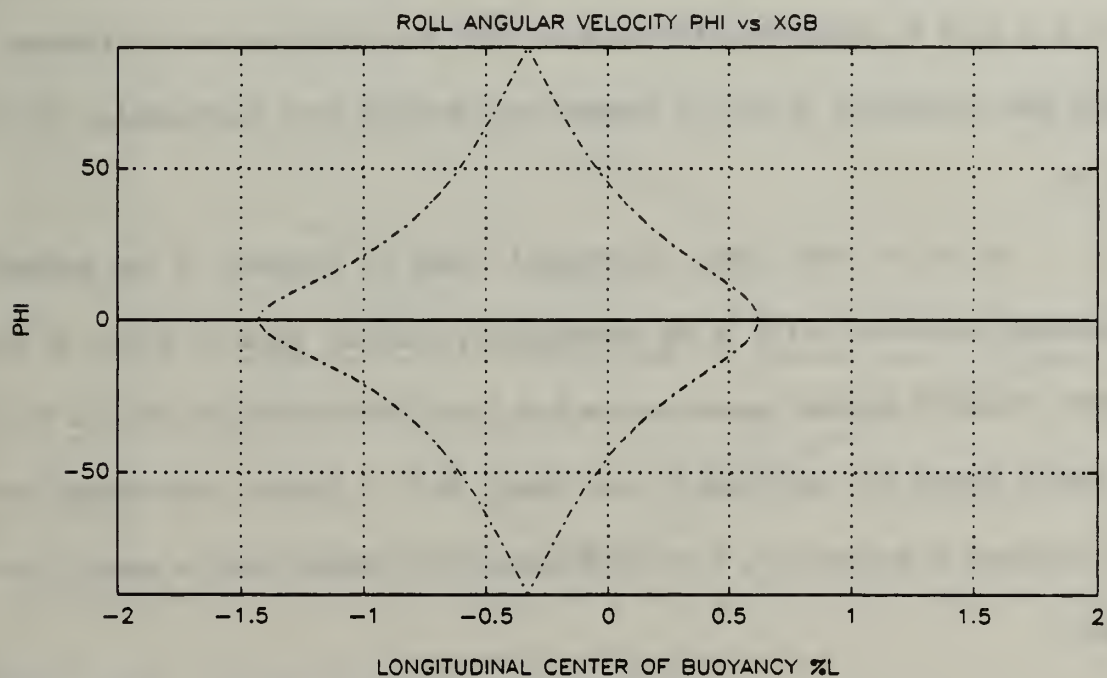


Figure 39. Steady State Roll Angle

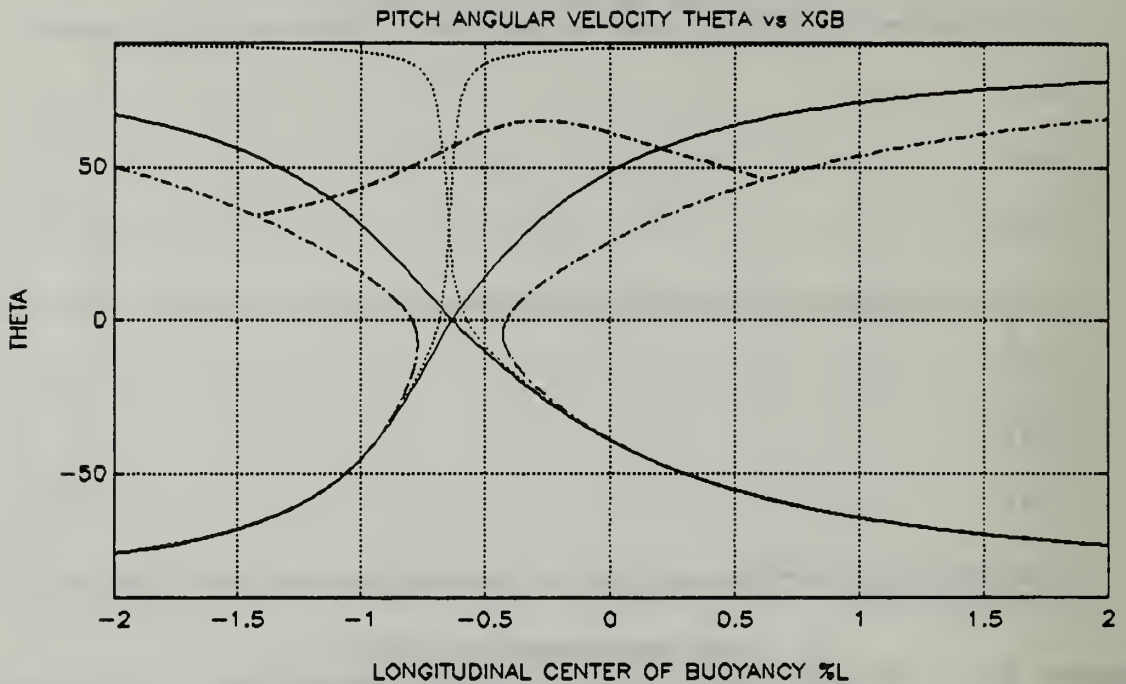


Figure 40. Steady State Pitch Angle

2. $\delta s = -20$ Degrees, δr is the Perturbation Parameter

Figures 41 through 48 show steady state solutions for the state variables u , v , w , p , q , r , Φ , θ . In Figures 41 through 48, solid lines correspond to $\delta r = 0$ degrees, dotted lines correspond to $\delta r = 2.5$ degrees, and dash-dot lines correspond to $\delta r = 5$ degrees.

In all of these cases, longitudinal center of buoyancy is the primary continuation parameter and δr is the perturbation parameter, while δs is kept at -20 degrees. Values of the other parameters are kept fixed: Excess buoyancy, $\delta B = 2\%$ of the vehicle weight (W): deflection of bow planes, $\delta b = 0$; location of horizontal and vertical centers of buoyancy, $x_B = z_B = 0$; location of the vertical center of gravity, $z_G = .1$ feet.

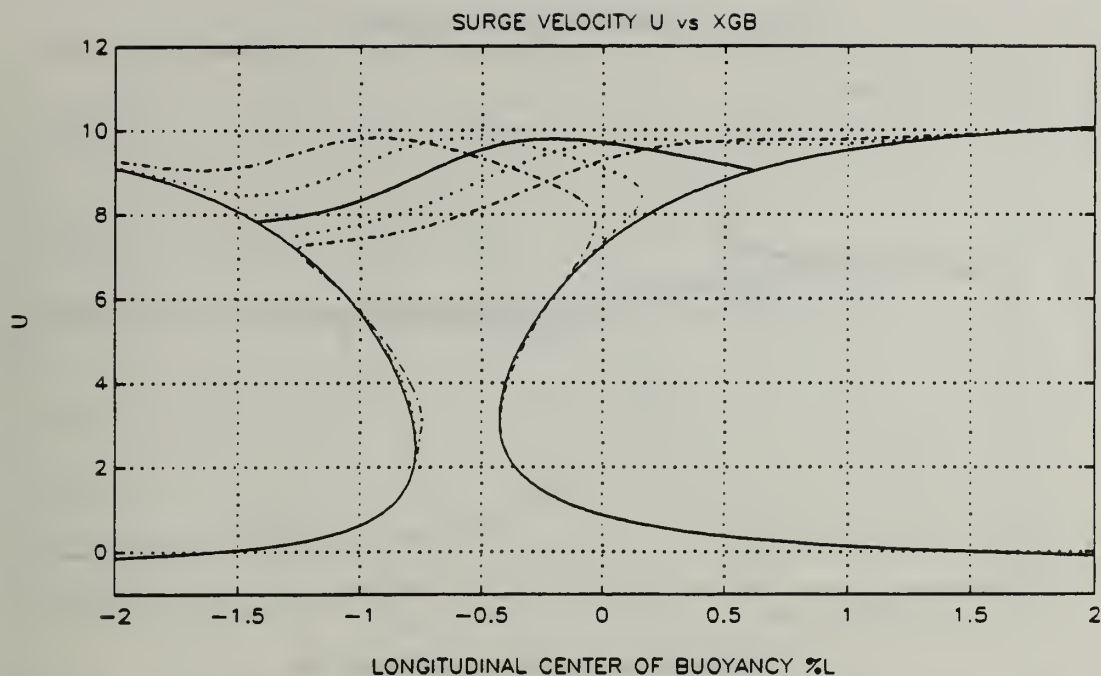


Figure 41. Steady State Surge Velocity

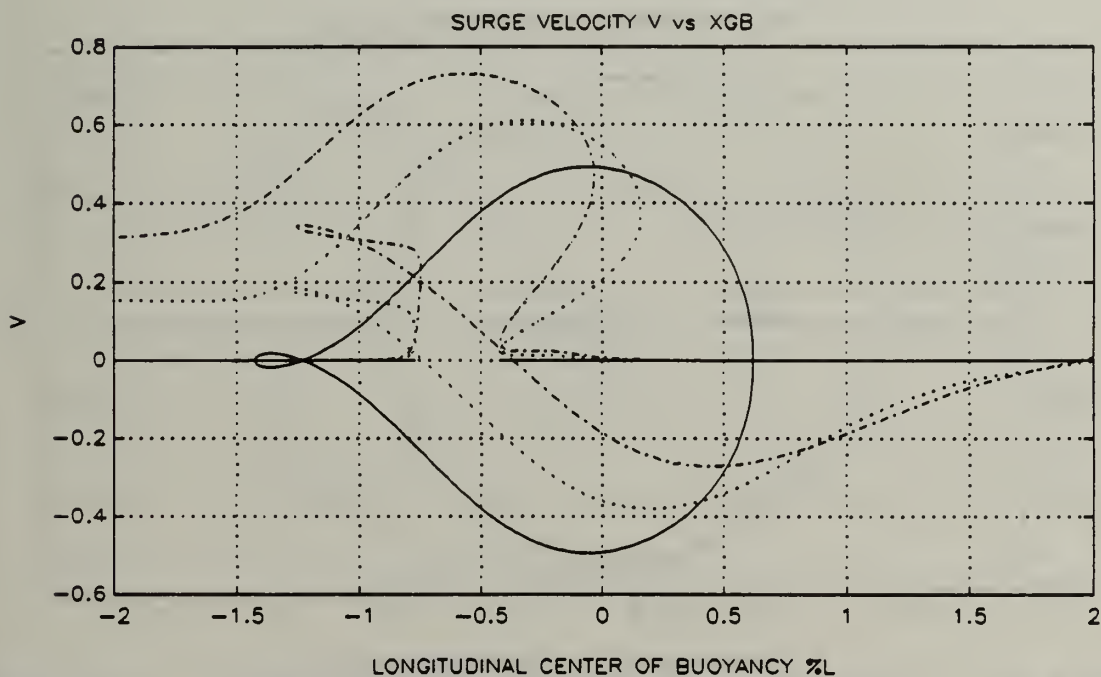


Figure 42. Steady State Sway Velocity

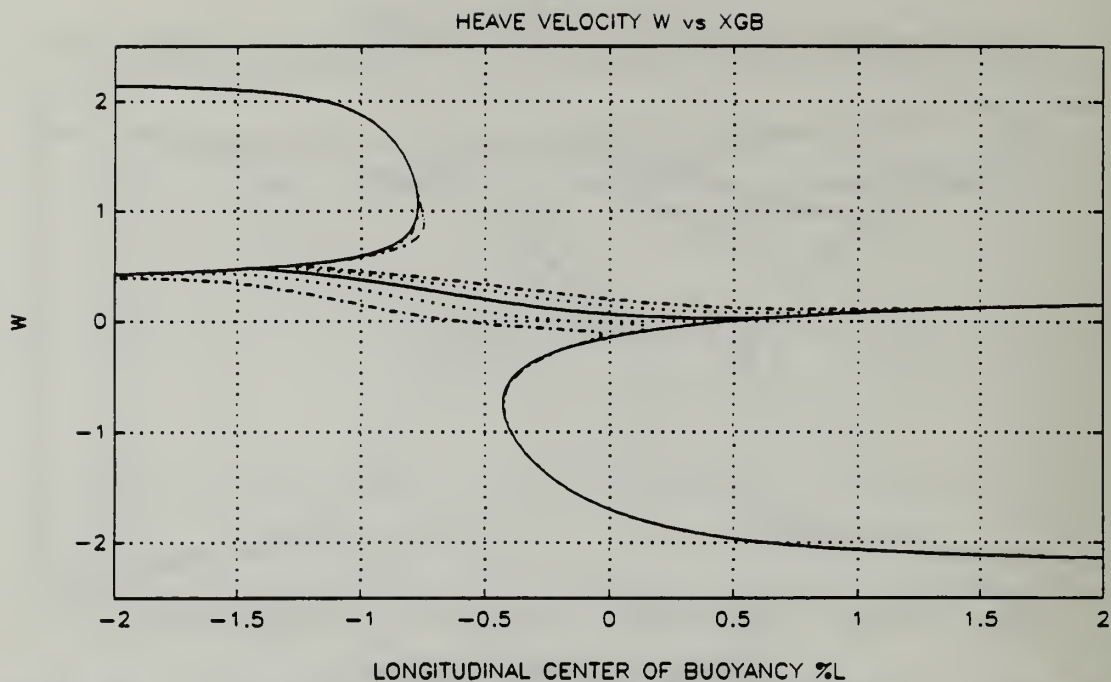


Figure 43. Steady State Heave Velocity

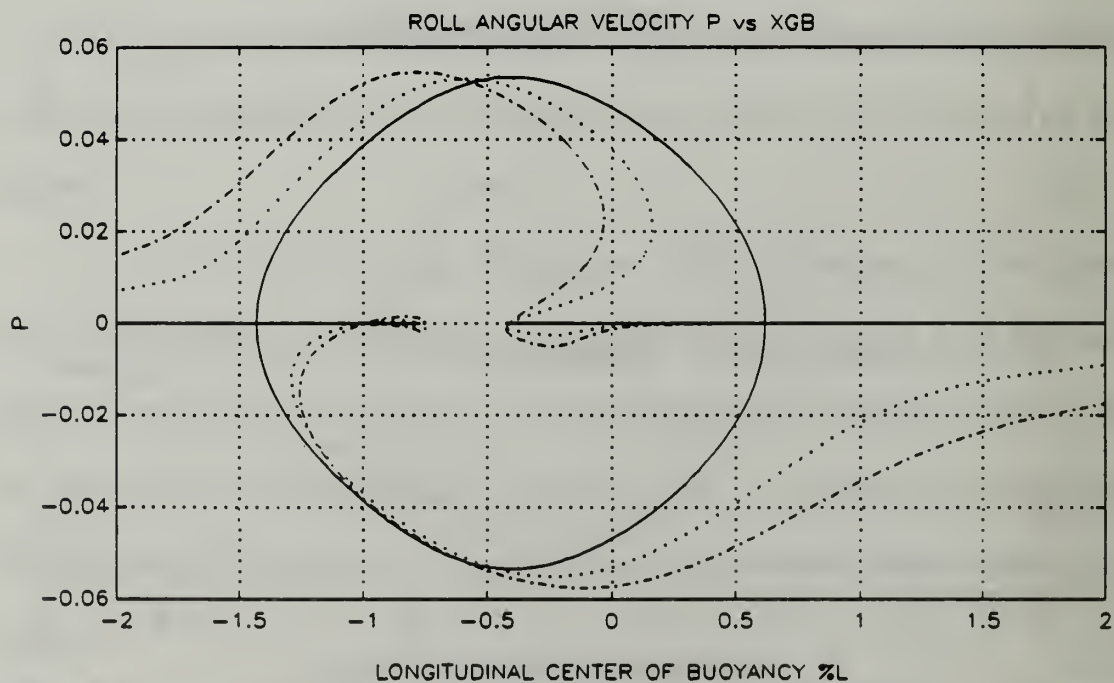


Figure 44. Steady State Roll Angular Velocity

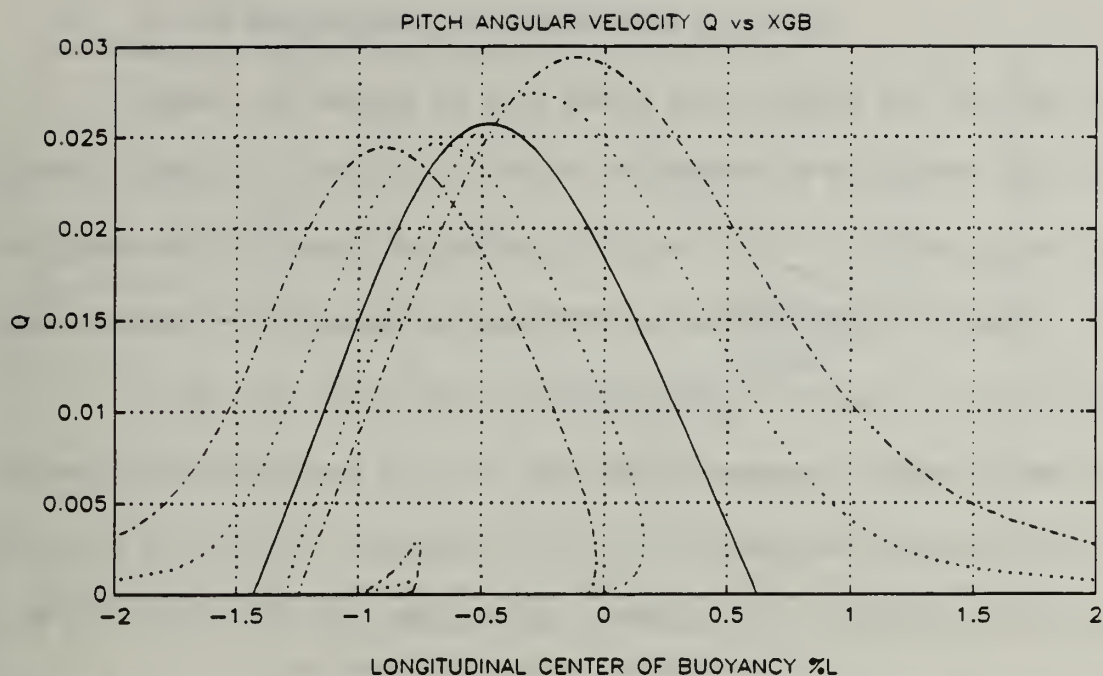


Figure 45. Steady State Yaw Angular Velocity

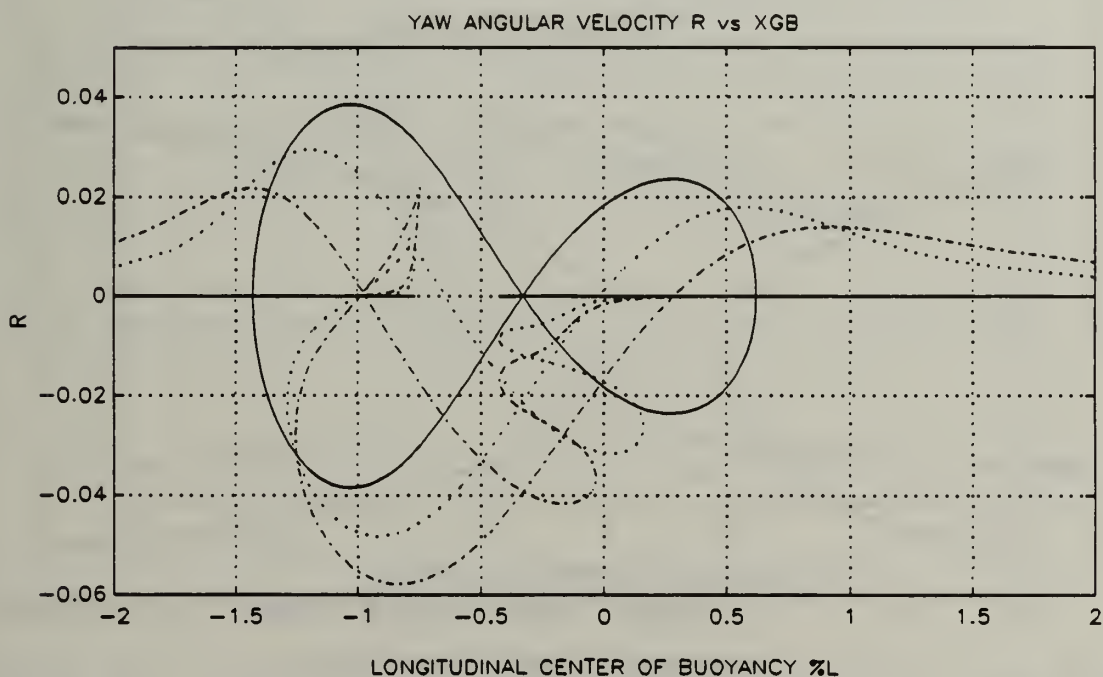


Figure 46. Steady State Yaw Angular Velocity

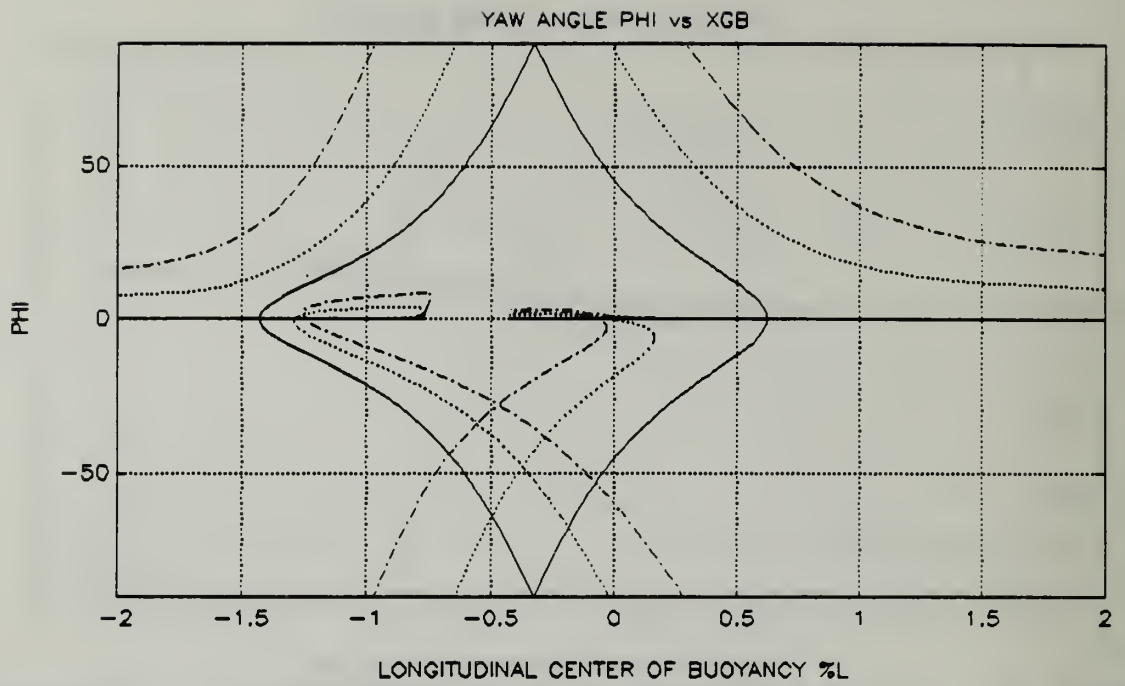


Figure 47. Steady State Roll Angle

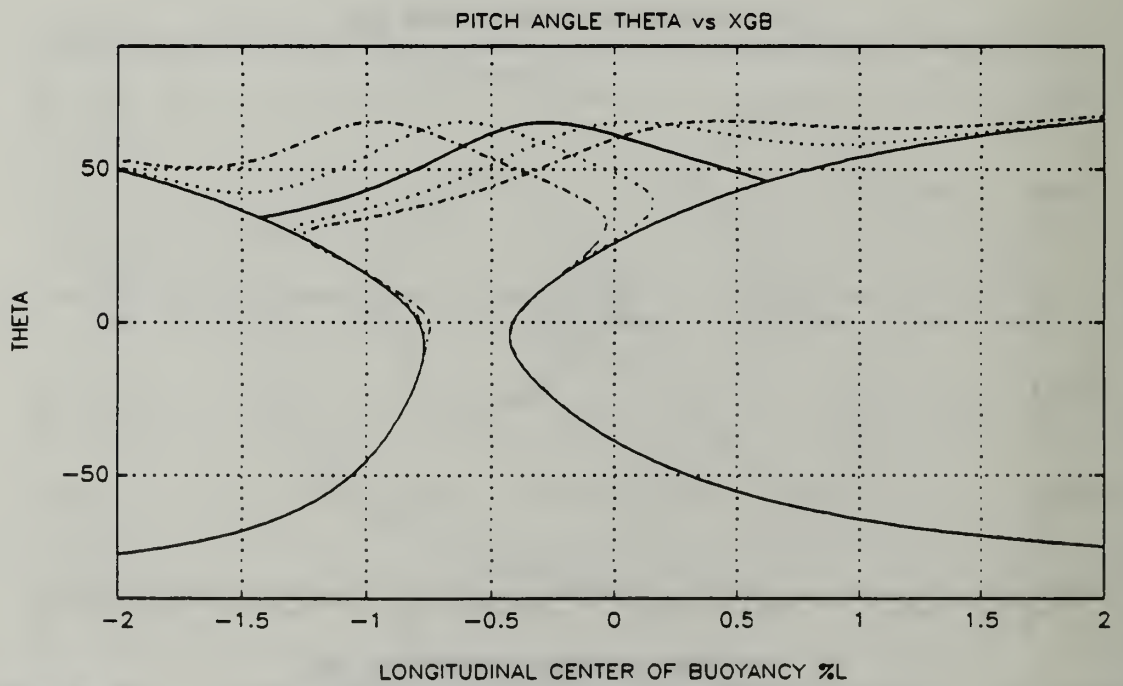


Figure 48. Steady State Pitch Angle

3. $\delta s = 0$ Degrees, δr is the Perturbation Parameter

Figures 49 through 56 show steady state solutions for the eight state variables, namely u , v , w , p , q , r , Φ , θ . In Figures 49 through 56, solid lines correspond to $\delta r = 0$ degrees, dashed lines correspond to $\delta r = 0.5$ degrees, dotted lines correspond to $\delta r = 2.5$ degrees and dash-dot lines correspond to $\delta r = 5$ degrees.

In all these cases, the longitudinal center of buoyancy is the primary continuation parameter and δr is the perturbation parameter. Values of the other parameters are kept fixed: Sternplane action, $\delta s = 0$ degrees; excess buoyancy, $\delta B = 2\%$ of the vehicle weight (W); deflection of bow planes, $\delta b = 0$; location of horizontal and vertical centers of buoyancy, $x_B = z_B = 0$; location of vertical center buoyancy, $z_{GB} = 0.1$ feet.

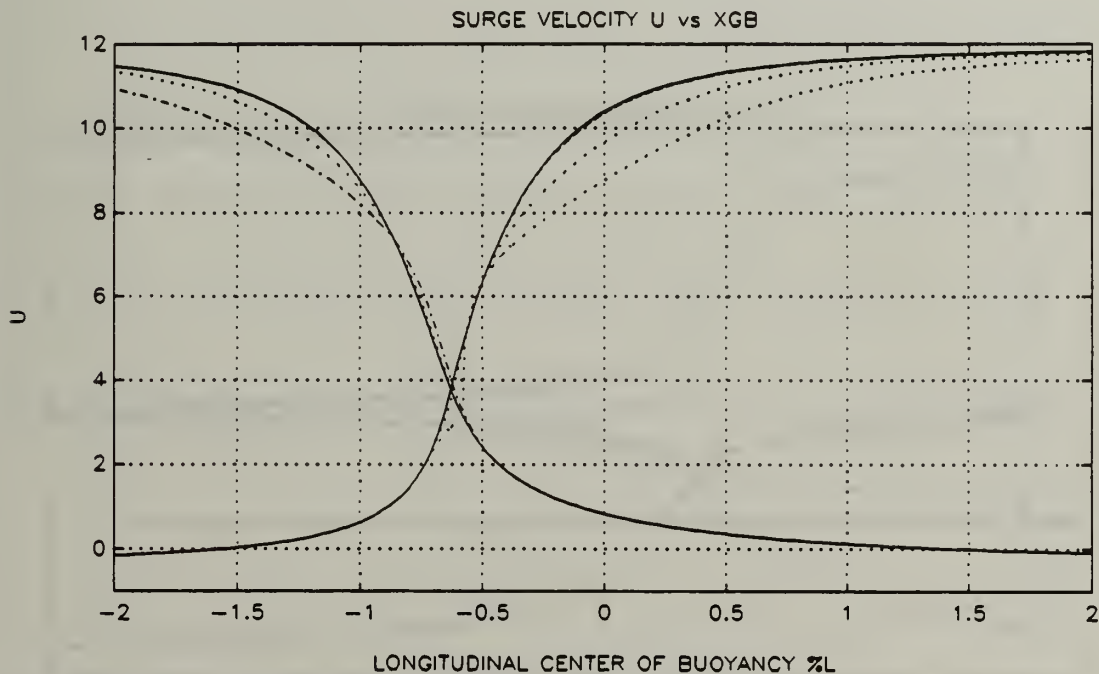


Figure 49. Steady State Surge Velocity

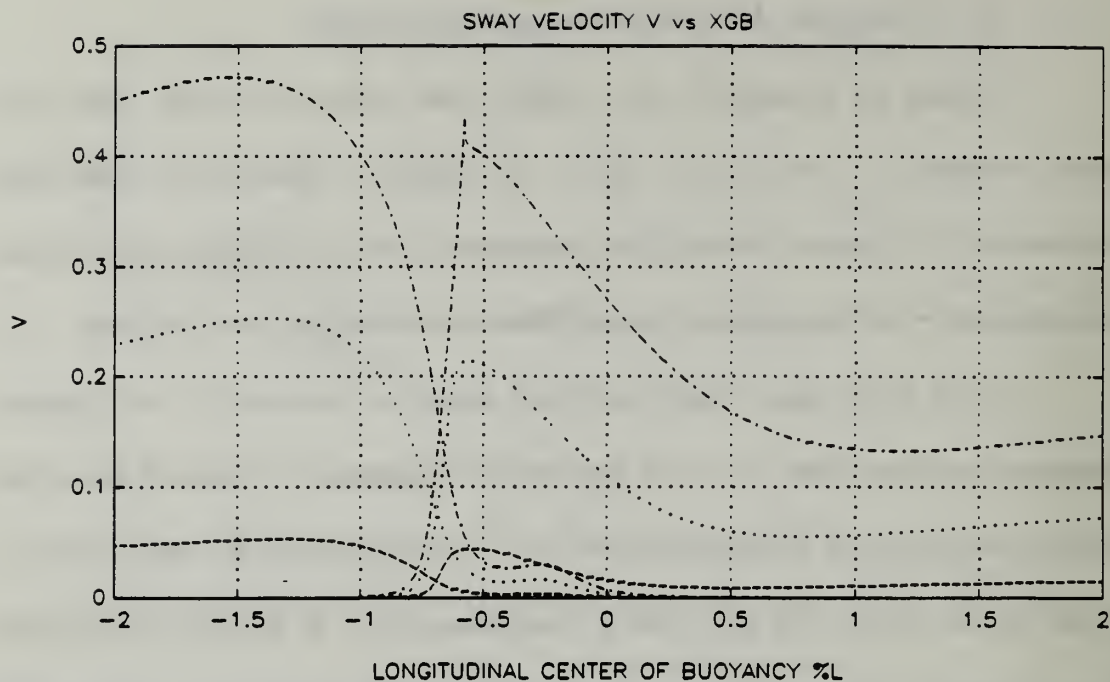


Figure 50. Steady State Sway Velocity

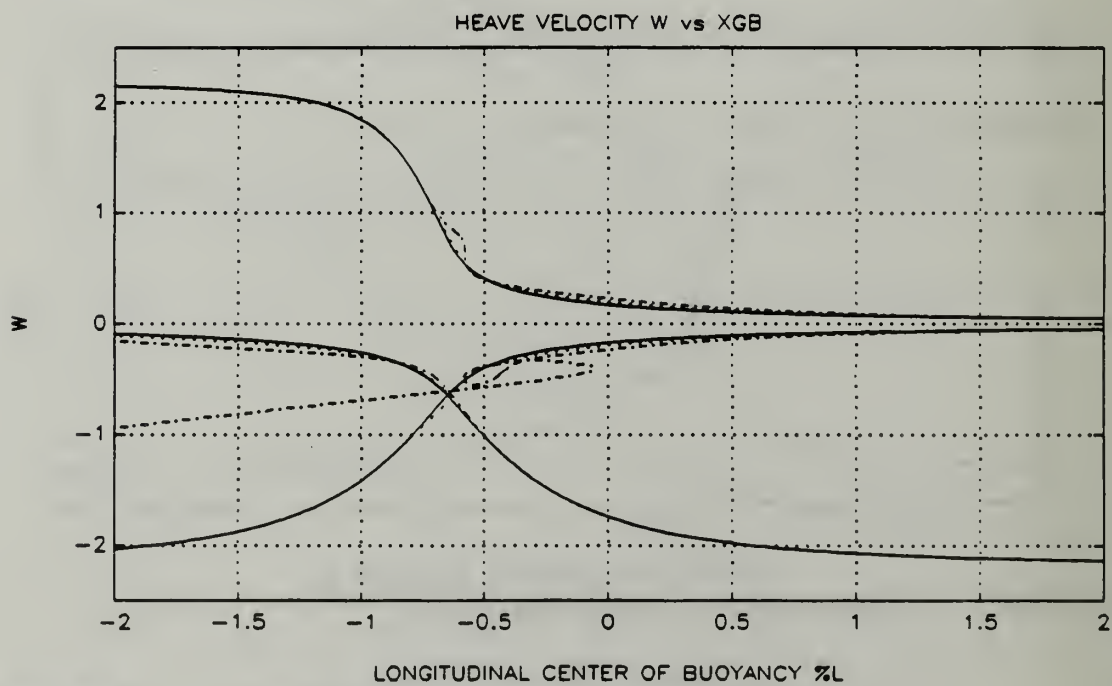


Figure 51. Steady State Heave Velocity

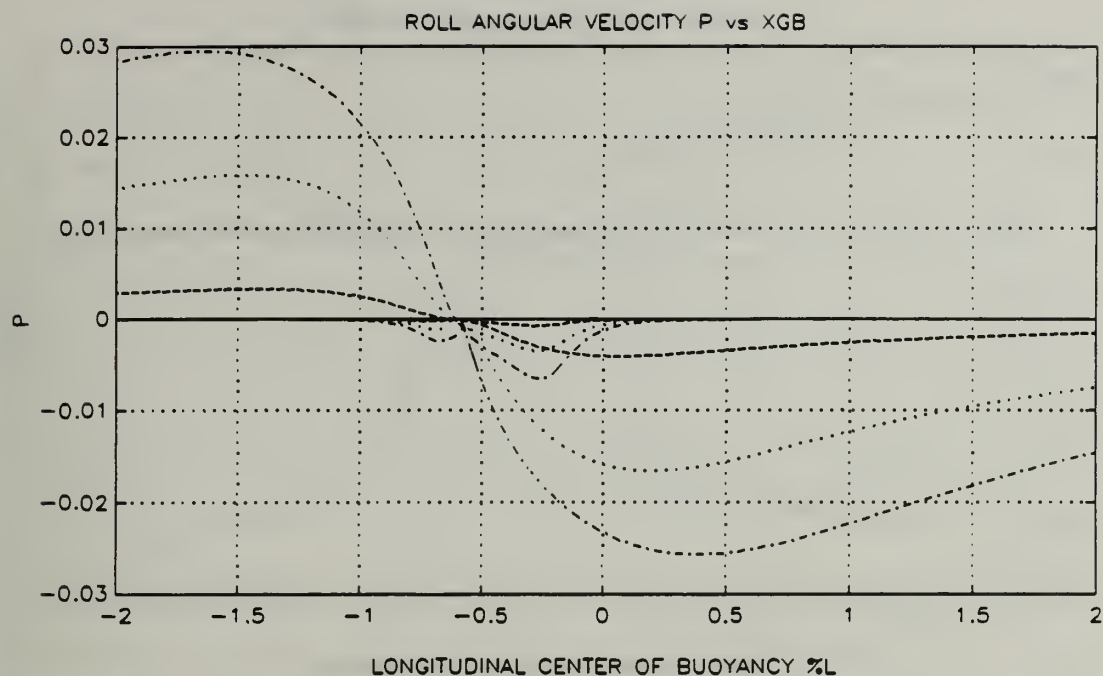


Figure 52. Steady State Roll Angular Velocity

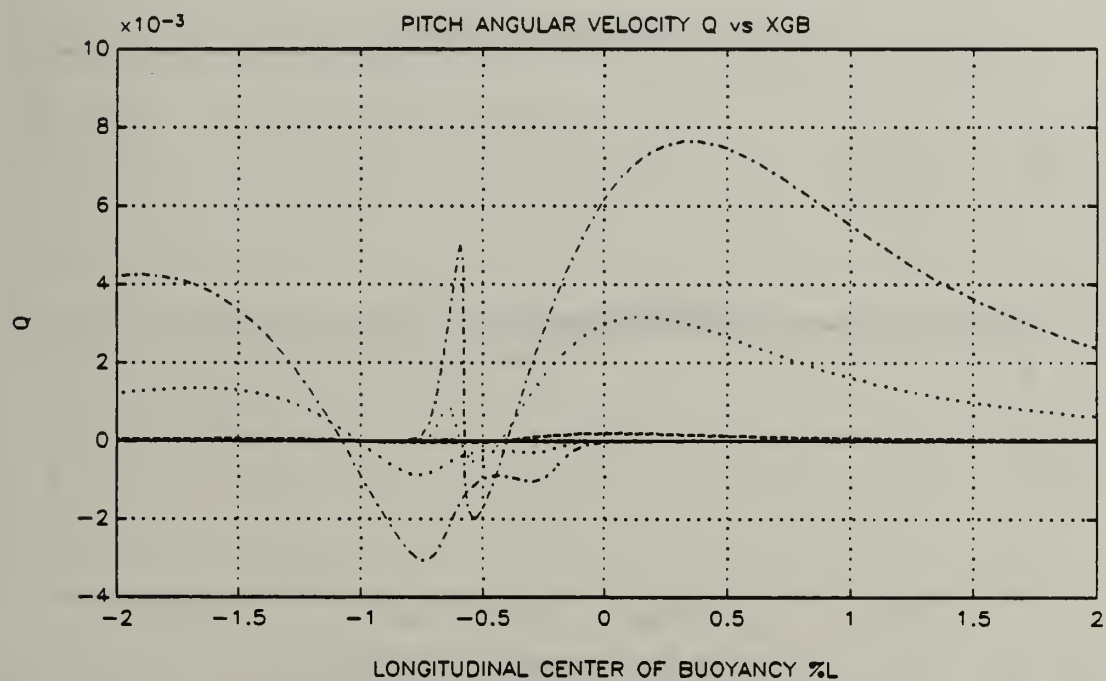


Figure 53. Steady State Pitch Angular Velocity

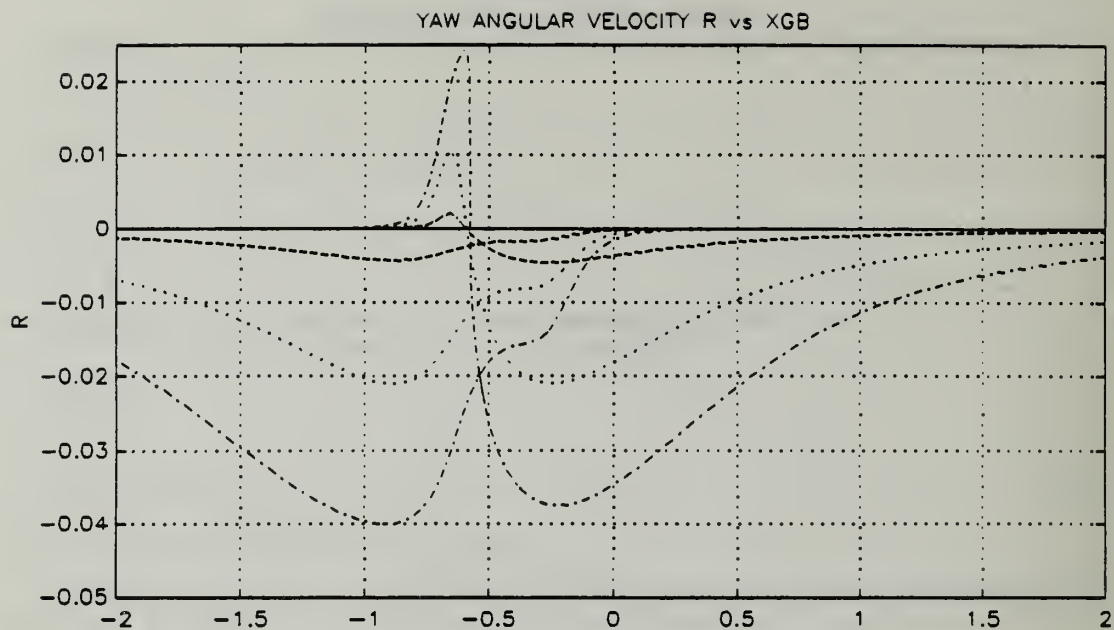


Figure 54. Steady State Yaw Angular Velocity

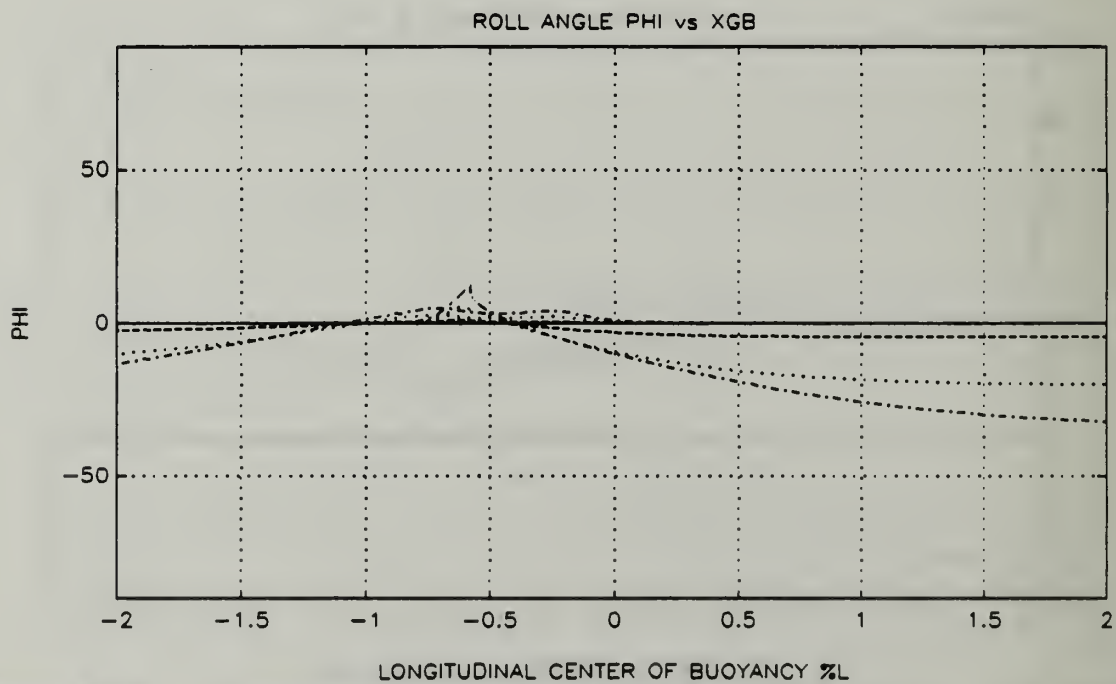


Figure 55. Steady State Roll Angle

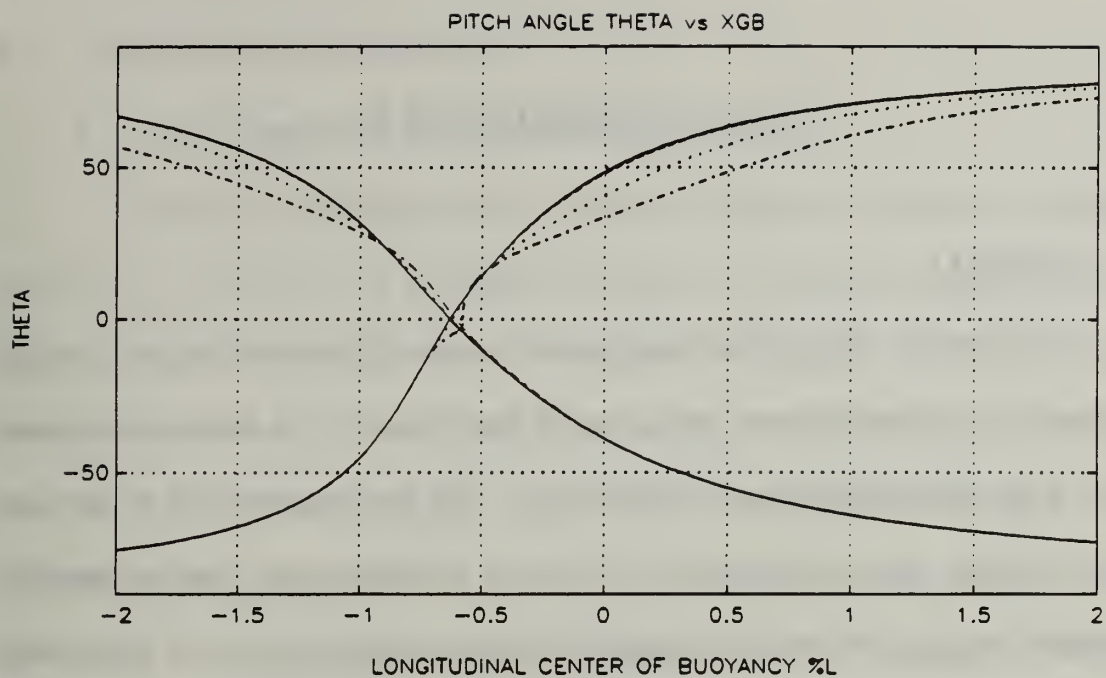


Figure 56. Steady State Pitch Angle

VI. CONTINUATION IN δB

A. GENERAL

In this chapter, the results of continuation in terms of the excess buoyancy will be discussed. In continuation runs, the system of equations (35) was solved for changing values of the primary parameter λ (in this case $\lambda = \delta B$). In these runs, first we fixed the value of rudder angle and changed the values of sternplane angle, then we kept the sternplane angle fixed at zero and changed the rudder angle to see how the steady state values were changing. When we kept the rudder angle at zero and changed the values of sternplane angle, we saw that there was a pitchfork bifurcation for the state variables v , p , r , and Φ . After we observed this pitchfork bifurcation for $\delta s = -20$ degrees and $\delta r = 0$ degrees, we started to change the rudder angle δr to see how the pitchfork bifurcation diagrams were changing. As an other continuation run we fixed $\delta r = 0$ degrees and changed δs towards positive values; however, we didn't see any branching in this case. As a last continuation run for this configuration, the value of sternplane angle was fixed at zero and the value of rudder angle was changed. We didn't see any branching behavior for this case.

B. STEADY STATE DIAGRAMS

1. $\delta r = 0$ Degrees, δs is the Perturbation Parameter

Figures 57 through 64 show steady state solutions for eight state variables, namely; u , v , w , p , q , r , Φ , θ . In Figures 57 through 64, solid lines correspond to $\delta s = 0$ degrees, dashed lines correspond to $\delta s = -5$ degrees, and dash-dot lines correspond to $\delta s = -20$ degrees. In all of these cases, excess buoyancy is the continuation parameter, while δr is kept at zero degrees. Values of the other parameters are kept fixed: longitudinal center of buoyancy $x_{GB} = -1$ % of the vehicles length (L); deflection of the bow planes, $\delta b = 0$; location of horizontal and vertical centers of buoyancy, $x_B = z_B = 0$; location of vertical center of buoyancy, $z_{GB} = .1$ feet.

As we can see from Figures 57 through 64, there is a pitchfork bifurcation for state variables u , v , p , r , Φ . We have out of plane solutions for sternplane angle $\delta s = -20$ degrees, this is also the configuration in which we observe bifurcations.

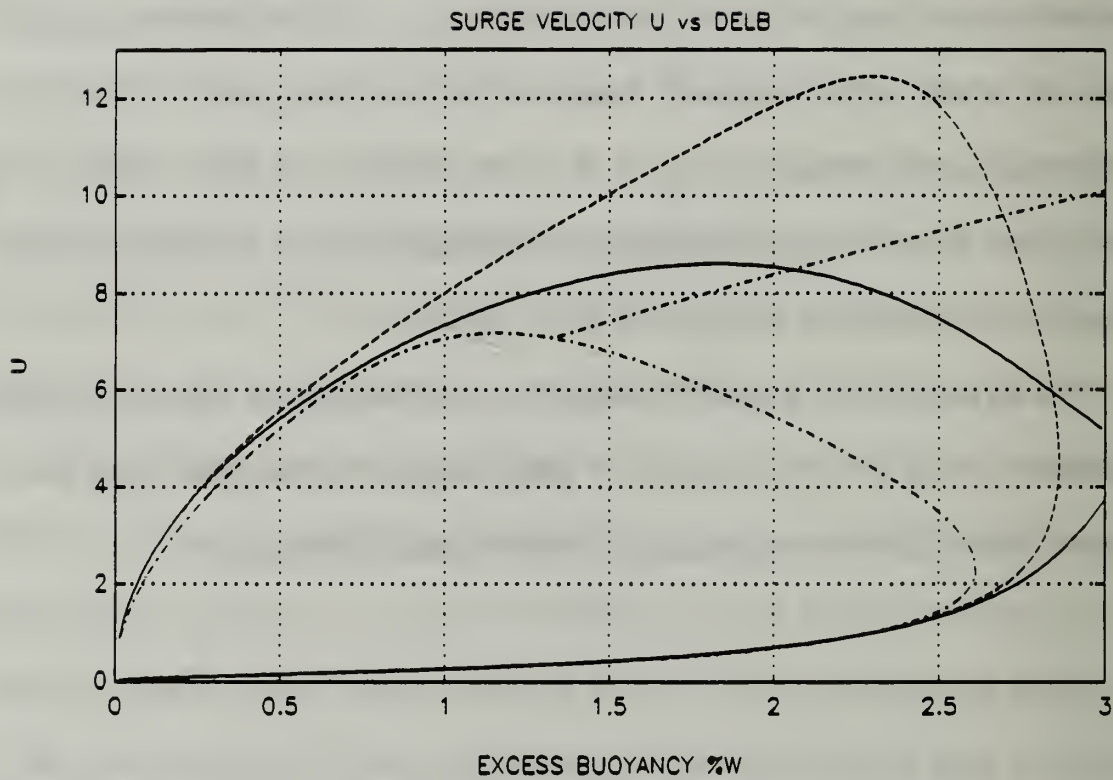


Figure 57. Steady State Surge Velocity

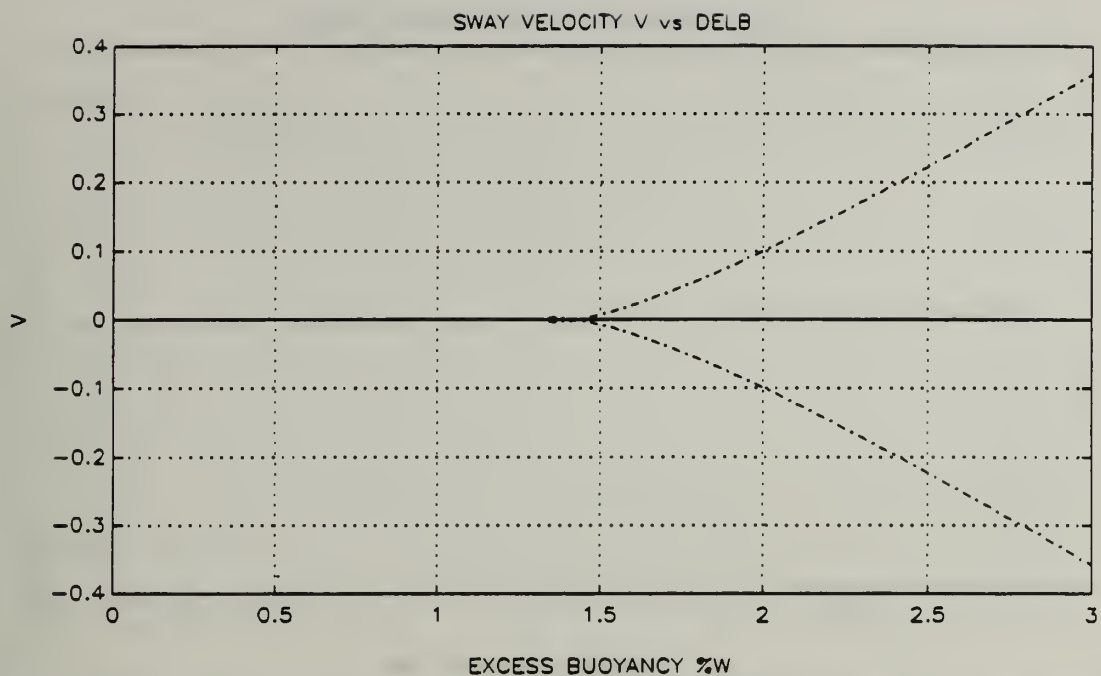


Figure 58. Steady State Sway Velocity

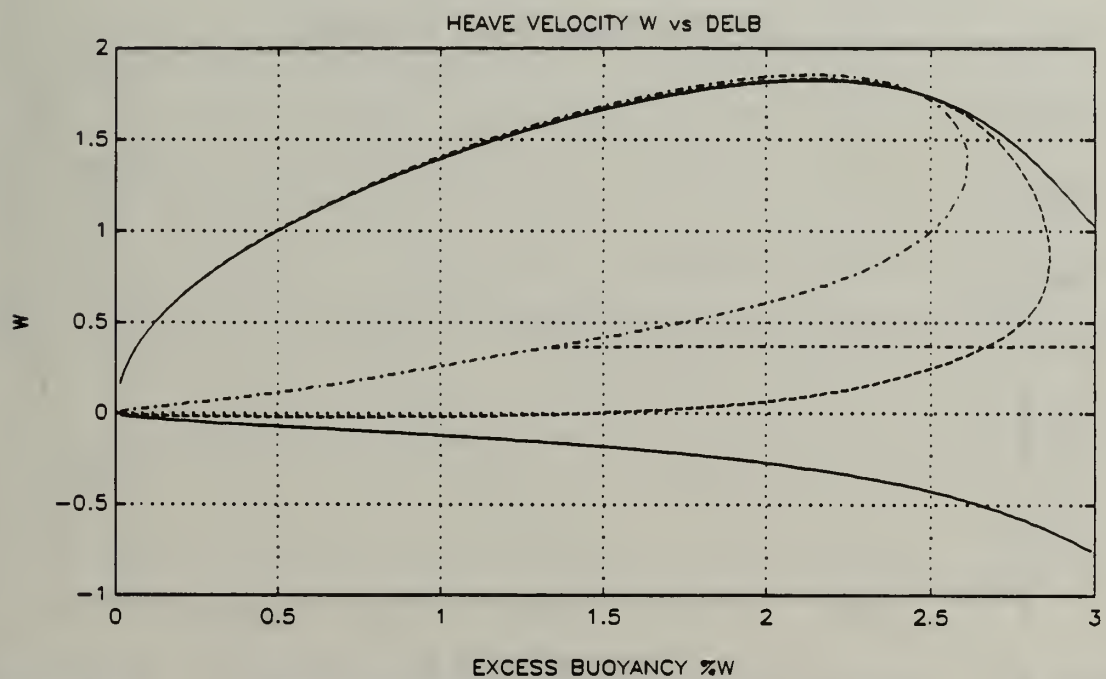


Figure 59. Steady State Heave Velocity

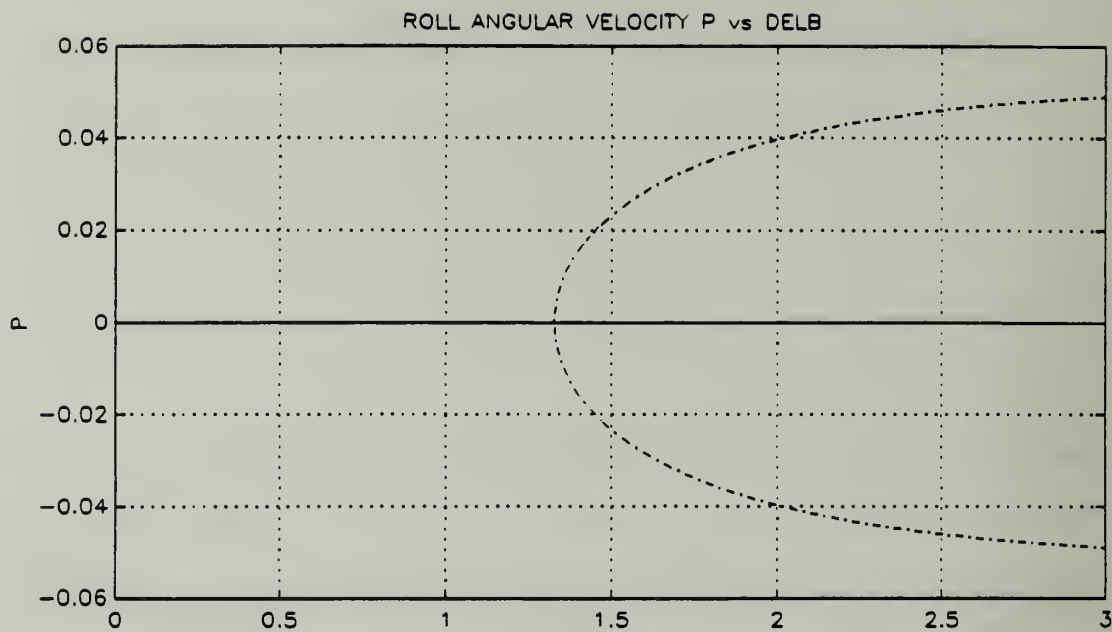


Figure 60. Steady State Roll Angular Velocity

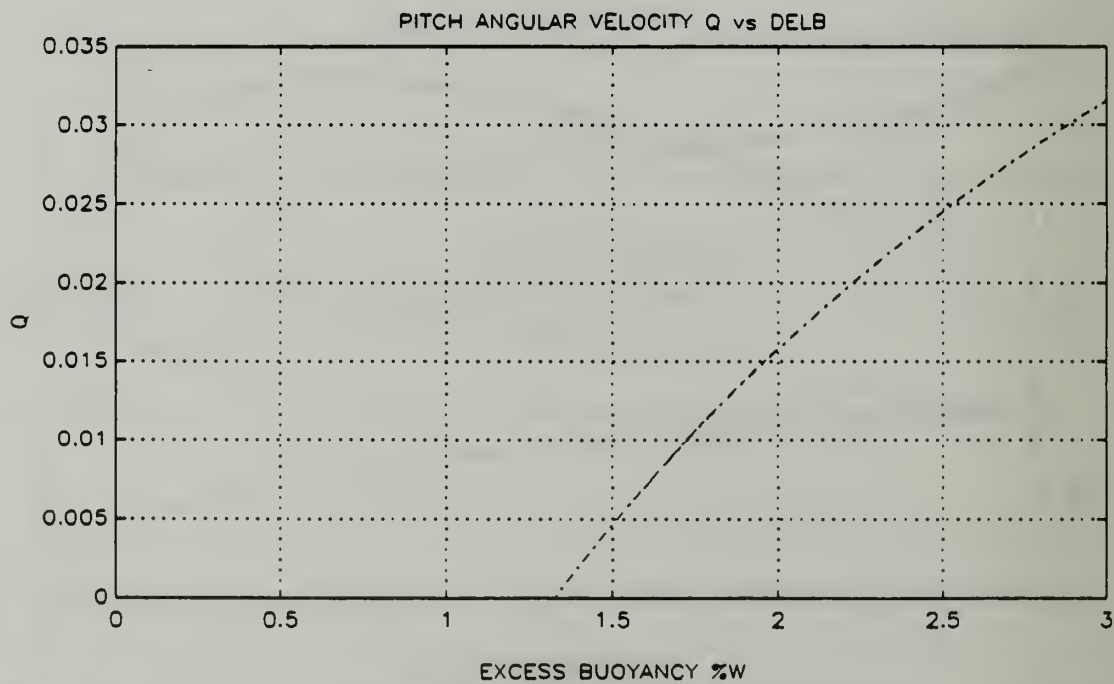


Figure 61. Steady State Pitch Angular Velocity

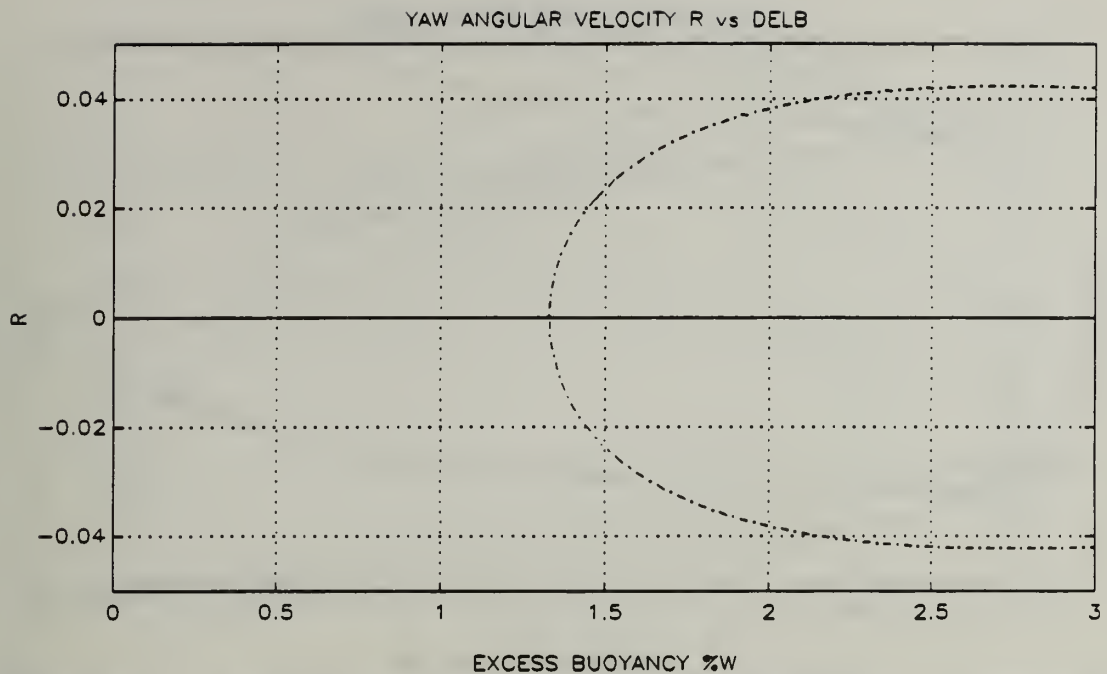


Figure 62. Steady State Yaw Angular Velocity

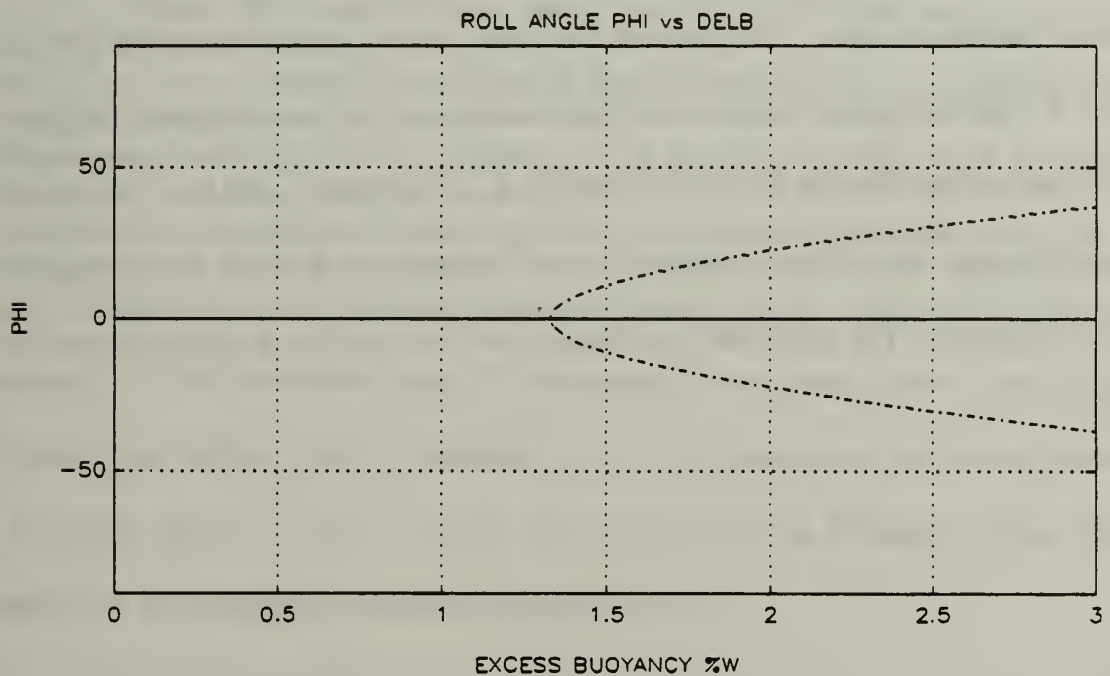


Figure 63. Steady State Roll Angle

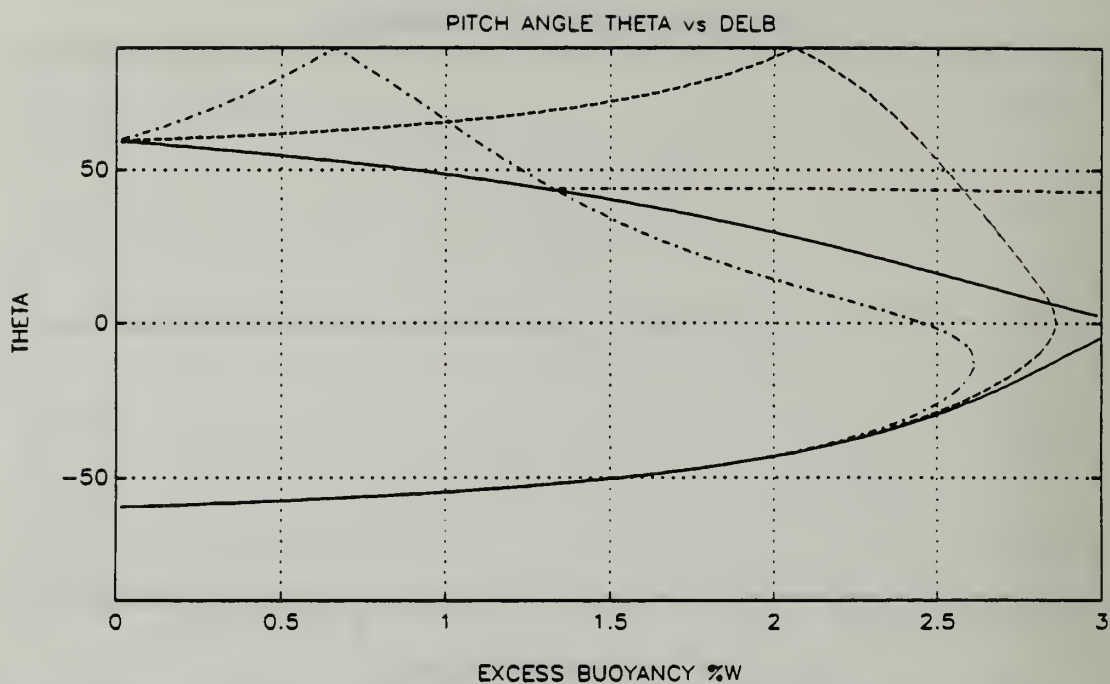


Figure 64. Steady State Pitch Angle

After analyzing the above diagrams, we decided to look at a case with a positive sternplane angle. We observed that there was no solution branching for this case. We got the inplane solutions and there were no out of plane solutions. In Figure 65, we can see the situation for the state variable u . In this figure, solid lines correspond to $\delta s = 0$ degrees, dotted lines correspond to $\delta s = 5$ degrees, and dash-dot lines correspond to $\delta s = 20$ degrees. The steady state solutions are not very sensitive to positive sternplane action.

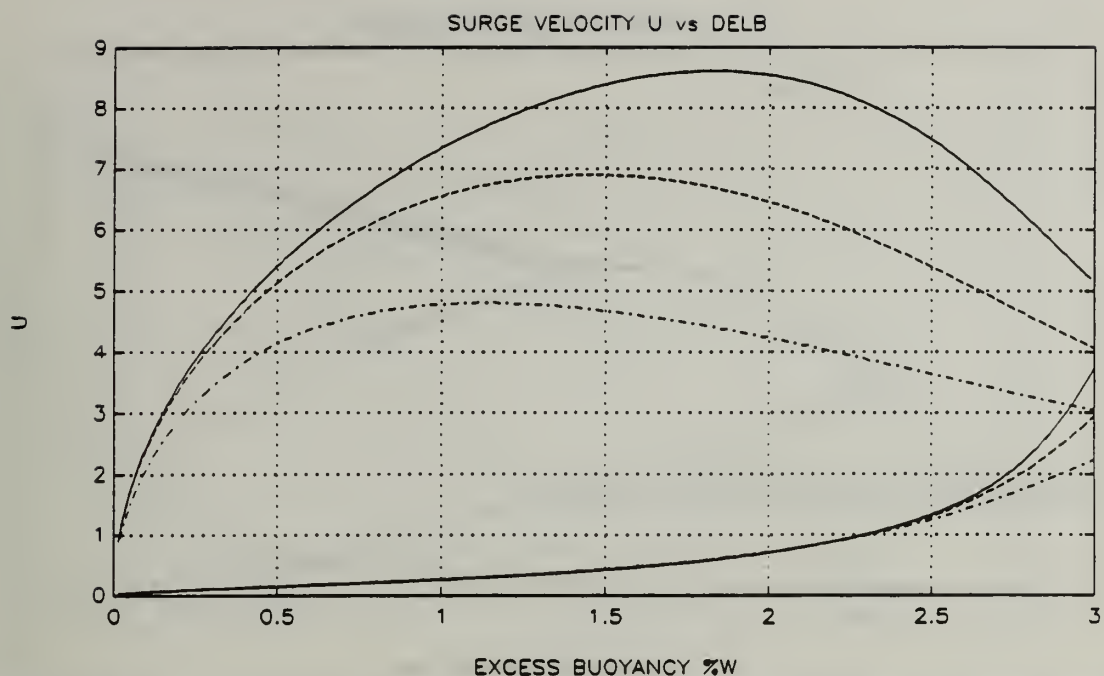


Figure 65. Steady State Surge Velocity When δs is Positive

2. $\delta s = -20$ Degrees, δr is the Perturbation Parameter

Figures 66 through 73 show steady state solutions for the state variables u , v , w , p , q , r , Φ , θ . In these figures, dashed lines correspond to $\delta r = 0.5$ degrees, and dash-dot lines correspond to $\delta r = 5$ degrees. In all of these the cases, excess buoyancy is the primary continuation parameter and δr is the perturbation parameter, while δs is kept -20 degrees. Values of the other parameters are kept fixed. Longitudinal center of buoyancy, -1 % of the vehicle length (L); deflection of bow planes, $\delta b = 0$; location of horizontal and vertical centers of buoyancy, $x_B = z_B = 0$; location of the vertical center of buoyancy, $z_{GB} = 0.1$ feet. By doing this, we saw how the bifurcation points were changing as we changed the perturbation parameter δr .

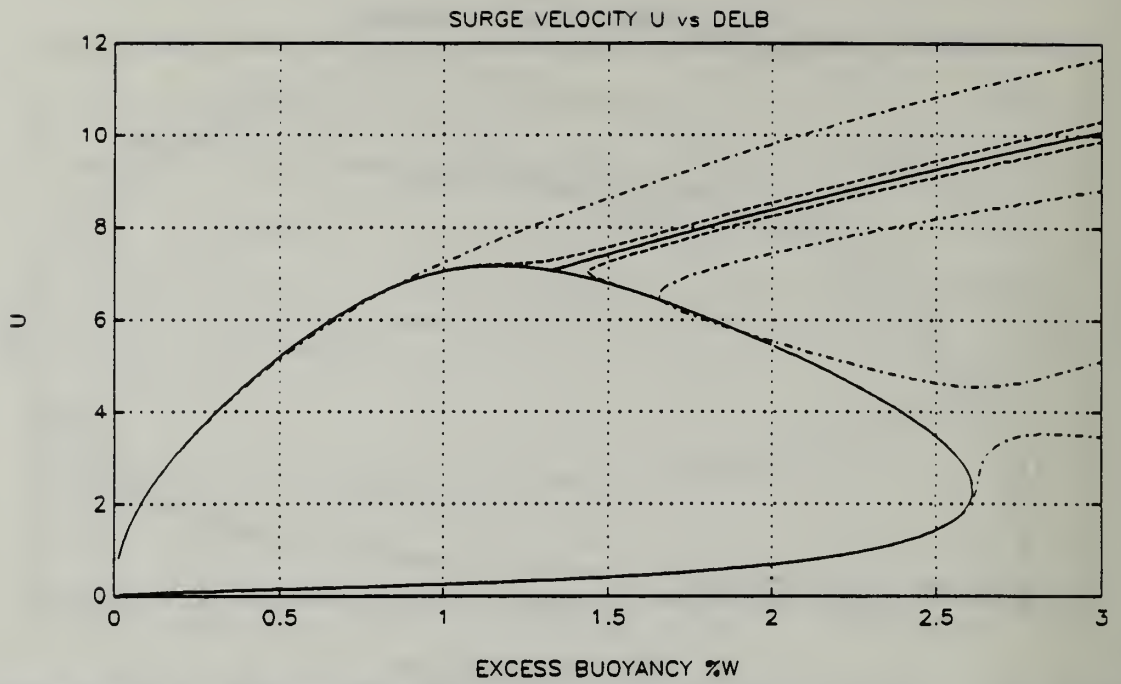


Figure 66. Steady State Surge Velocity

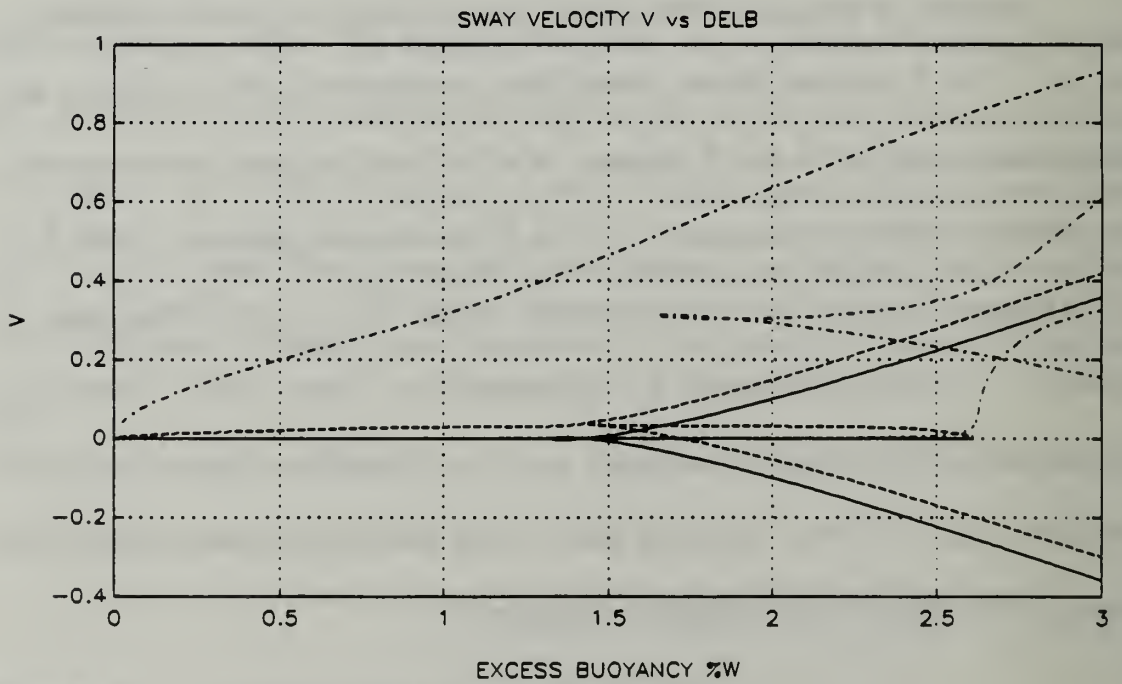


Figure 67. Steady State Sway Velocity

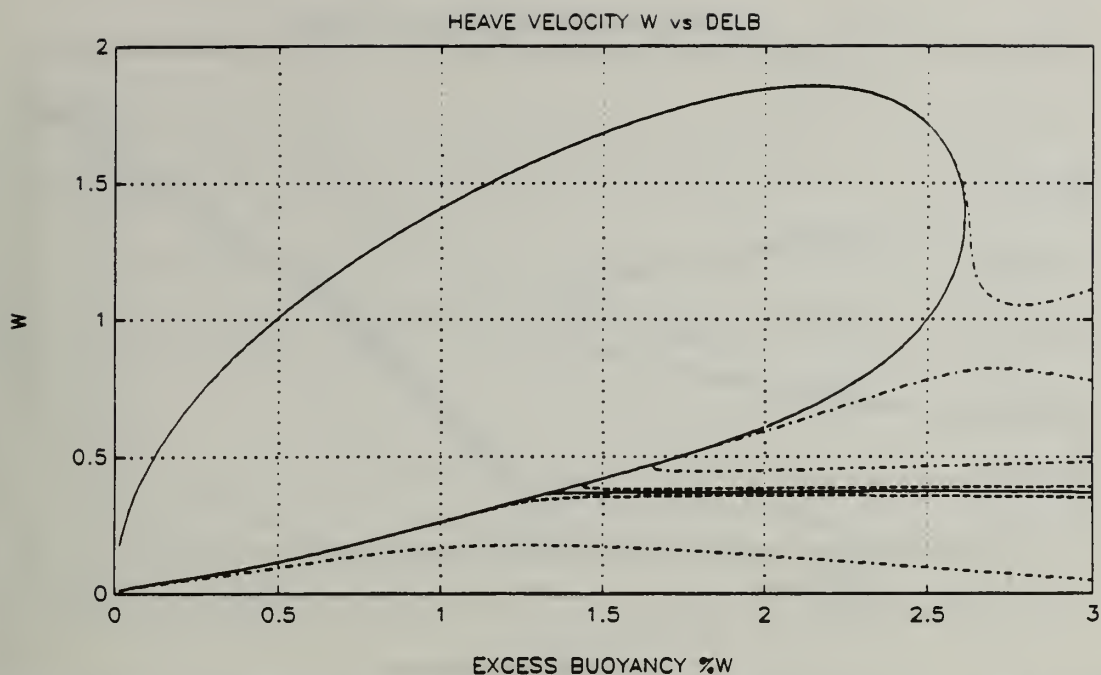


Figure 68. Steady State Heave Velocity

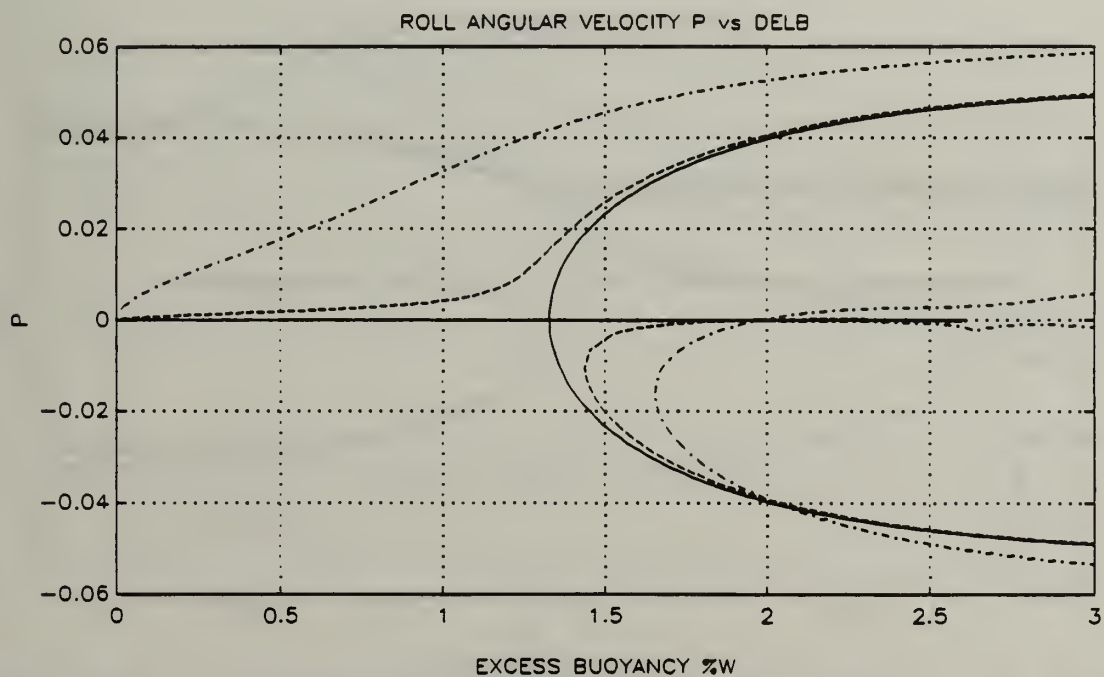


Figure 69. Steady State Roll Angular Velocity

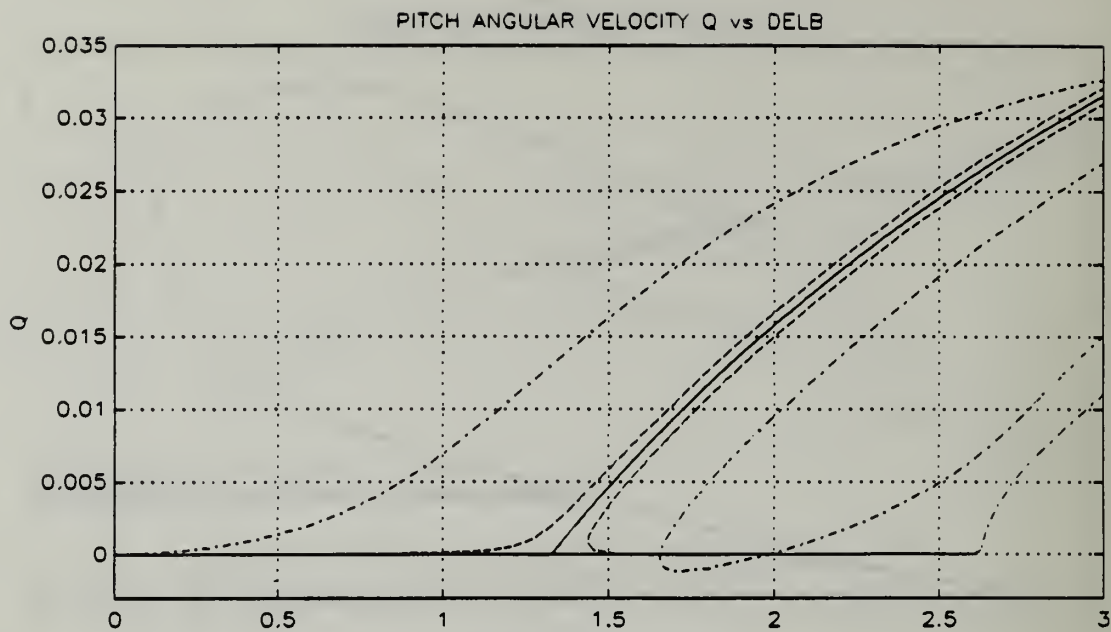


Figure 70. Steady State Pitch Angular Velocity

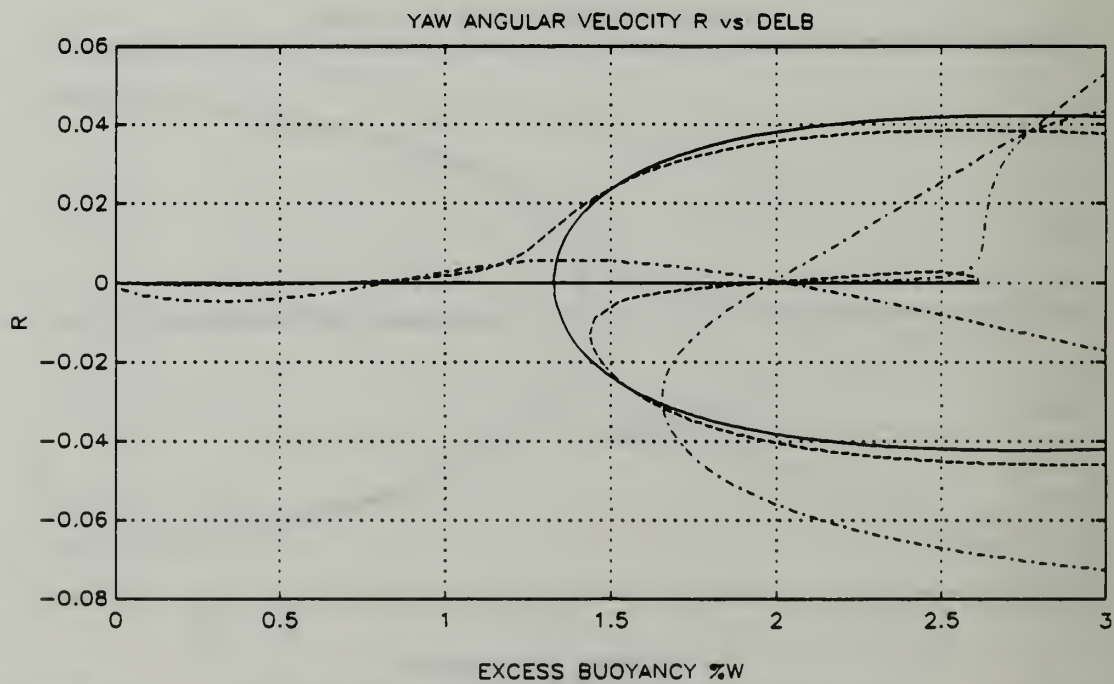


Figure 71. Steady State Yaw Angular Velocity

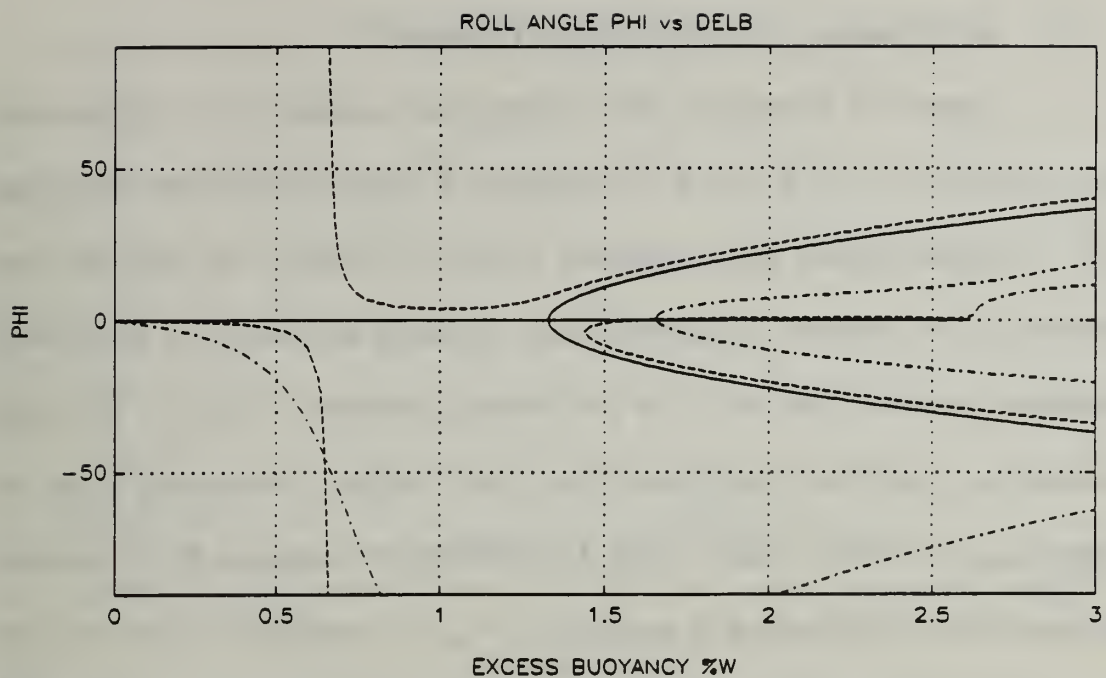


Figure 72. Steady State Roll Angle

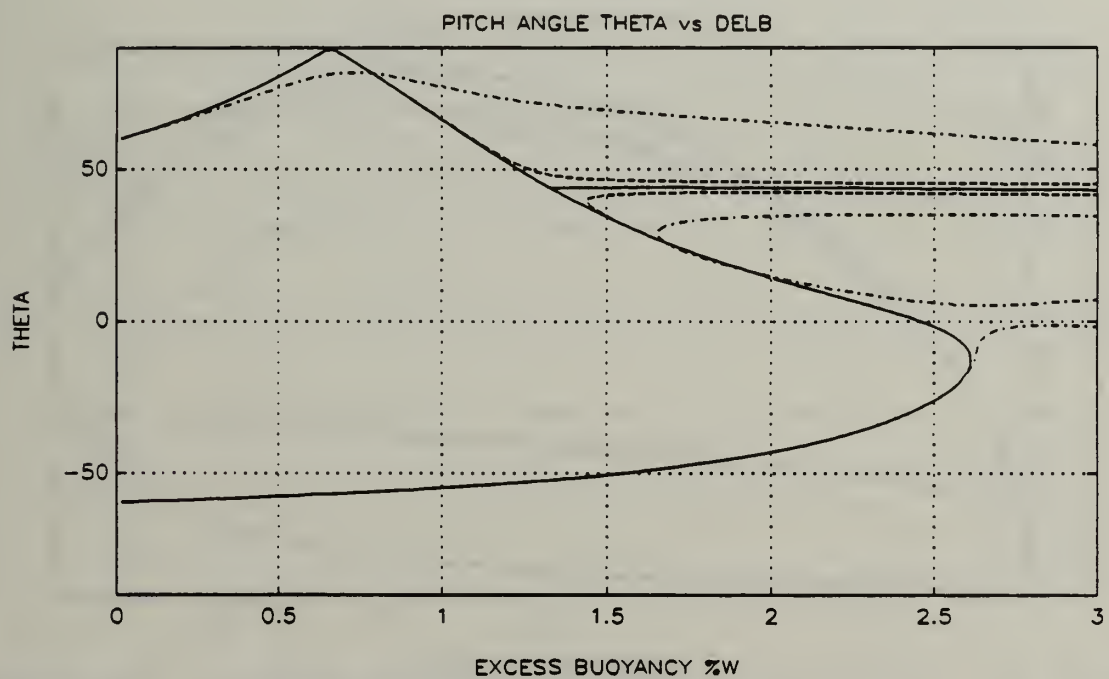


Figure 73. Steady State Pitch Angle

3. $\delta s=0$ Degrees, δr is Perturbation Parameter

Figures 74 through 81 show steady state solutions for the eight state variables, namely u , v , w , p , q , r , Φ , θ . In Figures 74 through 81 solid lines correspond to $\delta r=0$ degrees, dashed lines correspond to $\delta r=0.5$ degrees, and dash-dot lines correspond to $\delta r=5$ degrees. In all these cases, the excess buoyancy δB is the primary continuation parameter and δr is the perturbation parameter. Values of the other parameters are kept fixed: Sternplane action, $\delta s=0$ degrees; longitudinal center of buoyancy $x_{GB}=-1\%$ of the vehicle length (L); deflection of bowplanes, $\delta b=0$; location of horizontal and vertical centers of buoyancy, $x_B = z_B = 0$; location of vertical center of buoyancy, $z_{GB} = 0.1$ feet. As we can see, there was no solution branching for this case. Of course, we got out of plane solutions for nonzero rudder angle values.

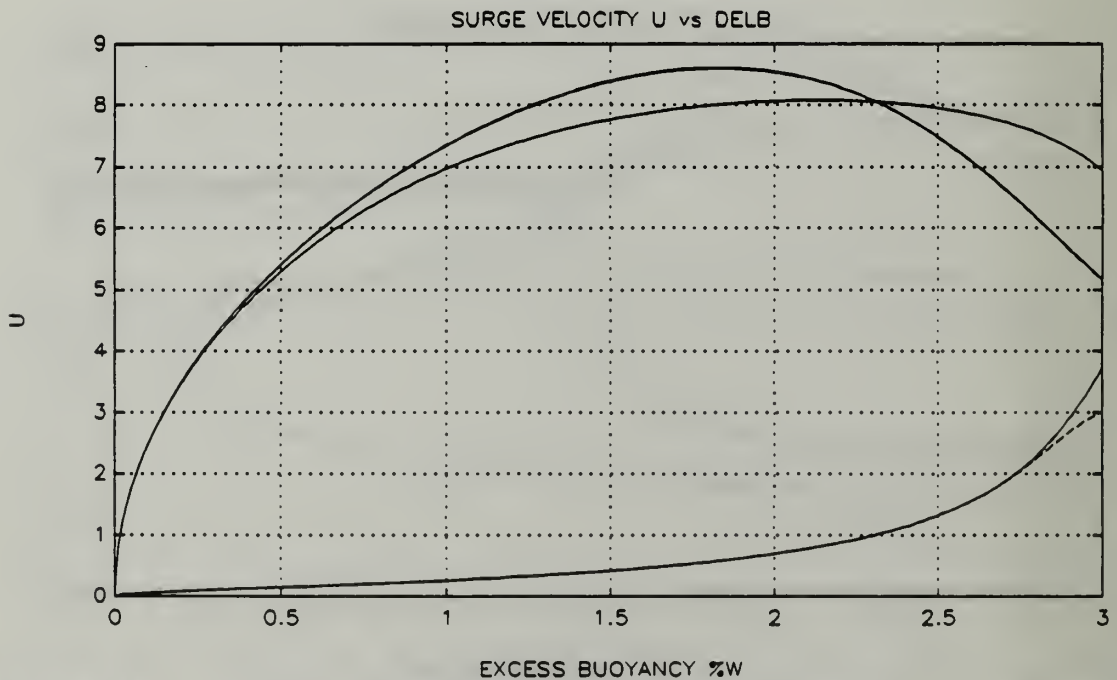


Figure 74. Steady State Surge Velocity

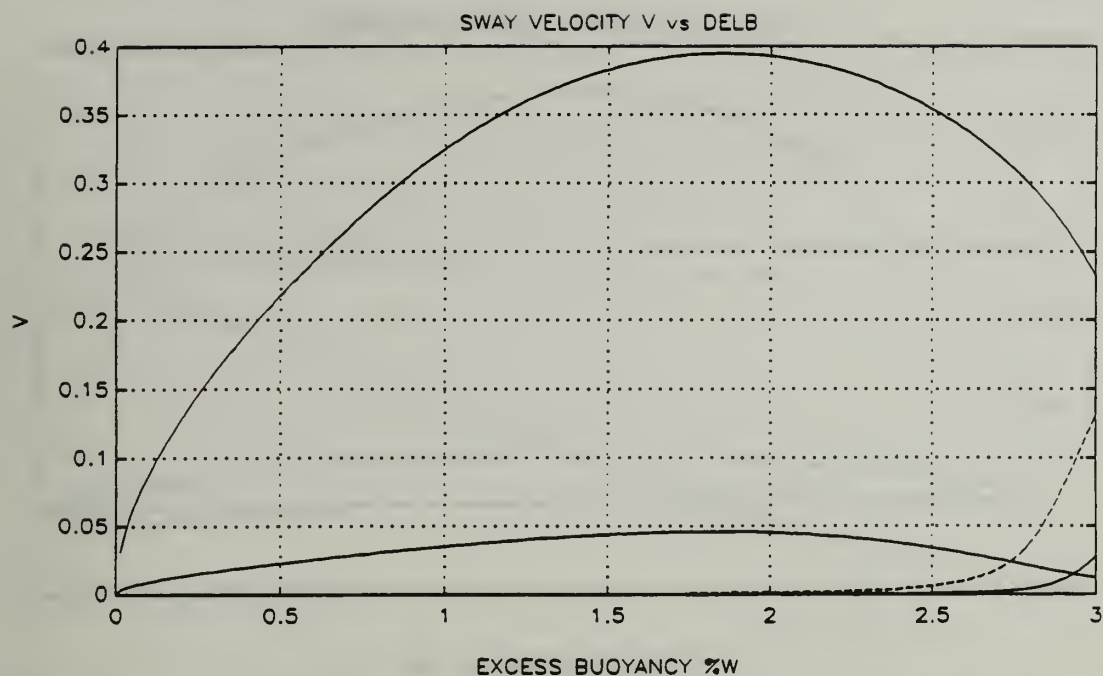


Figure 75. Steady State Sway Velocity

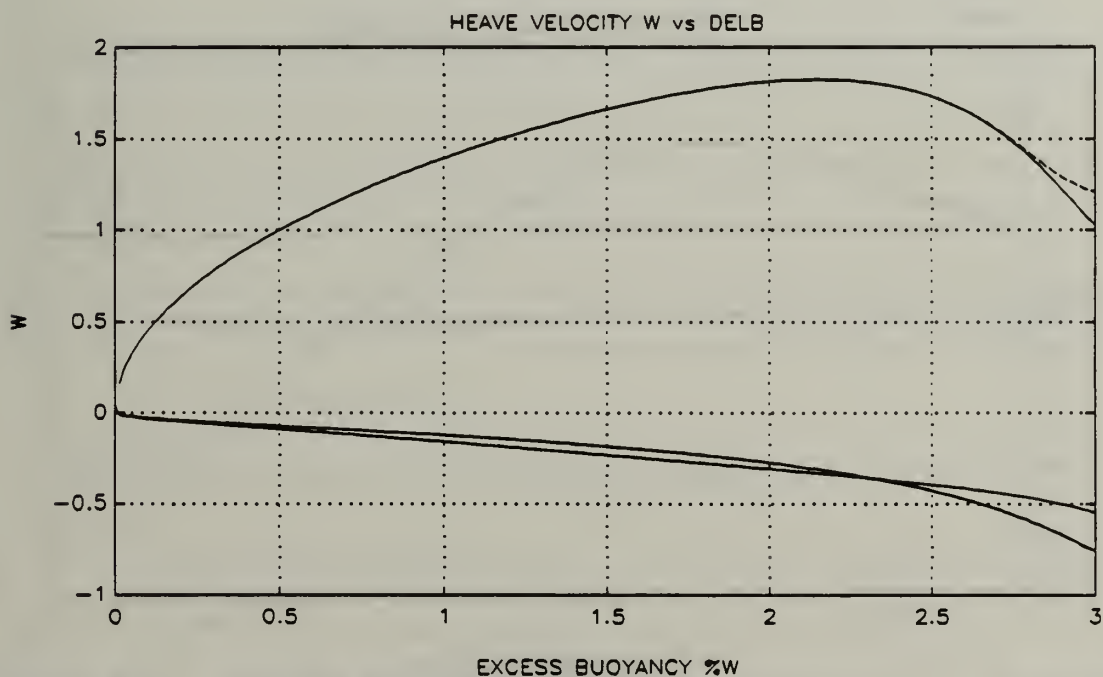


Figure 76. Steady State Heave Velocity

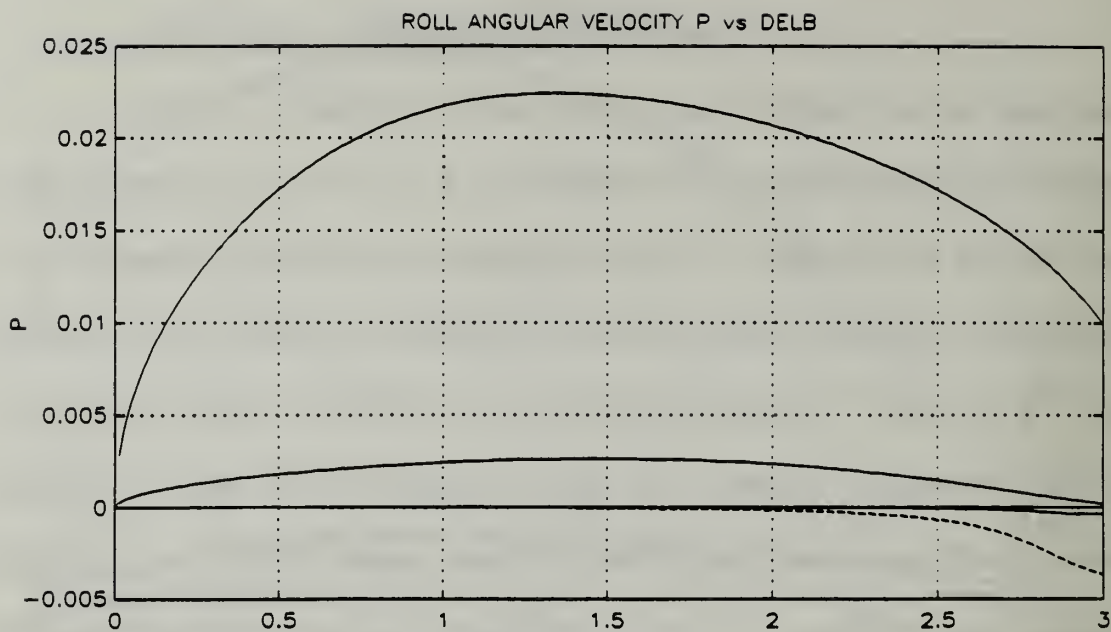


Figure 77. Steady State Pitch Angular Velocity

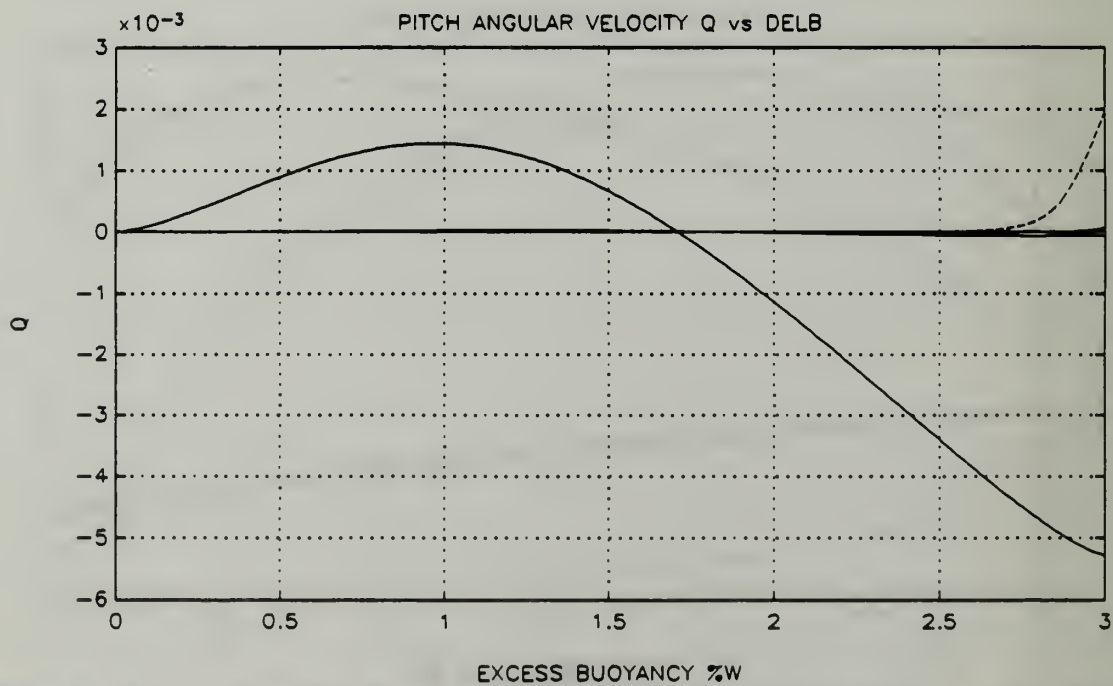


Figure 78. Steady State Roll Angle

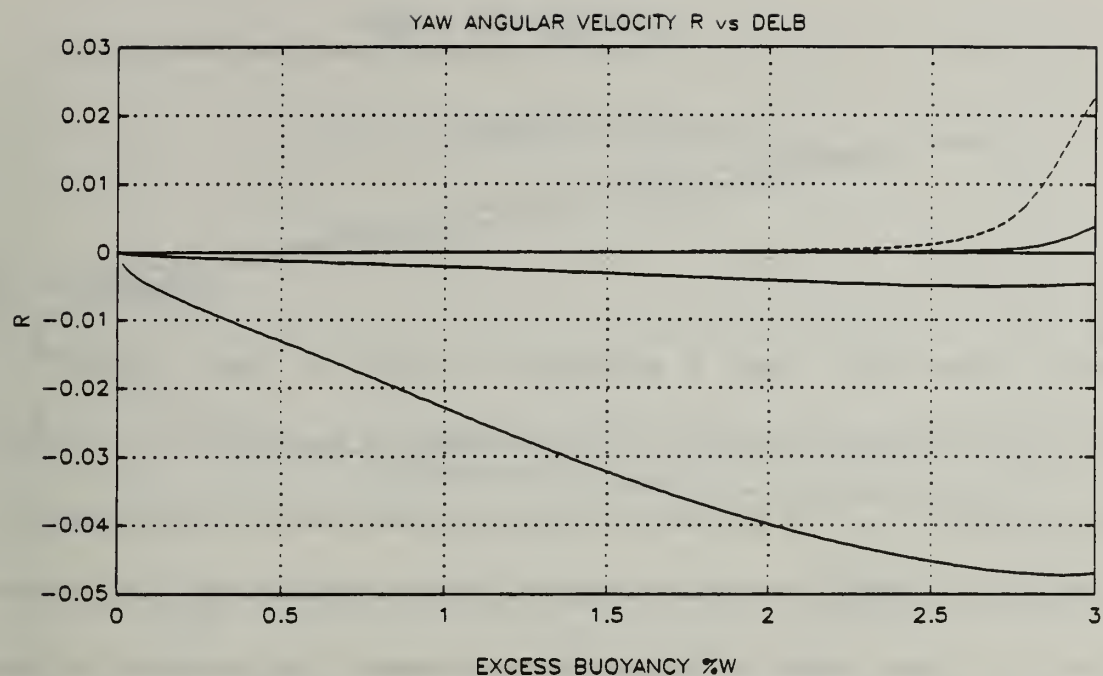


Figure 79. Steady State Yaw Angular Velocity

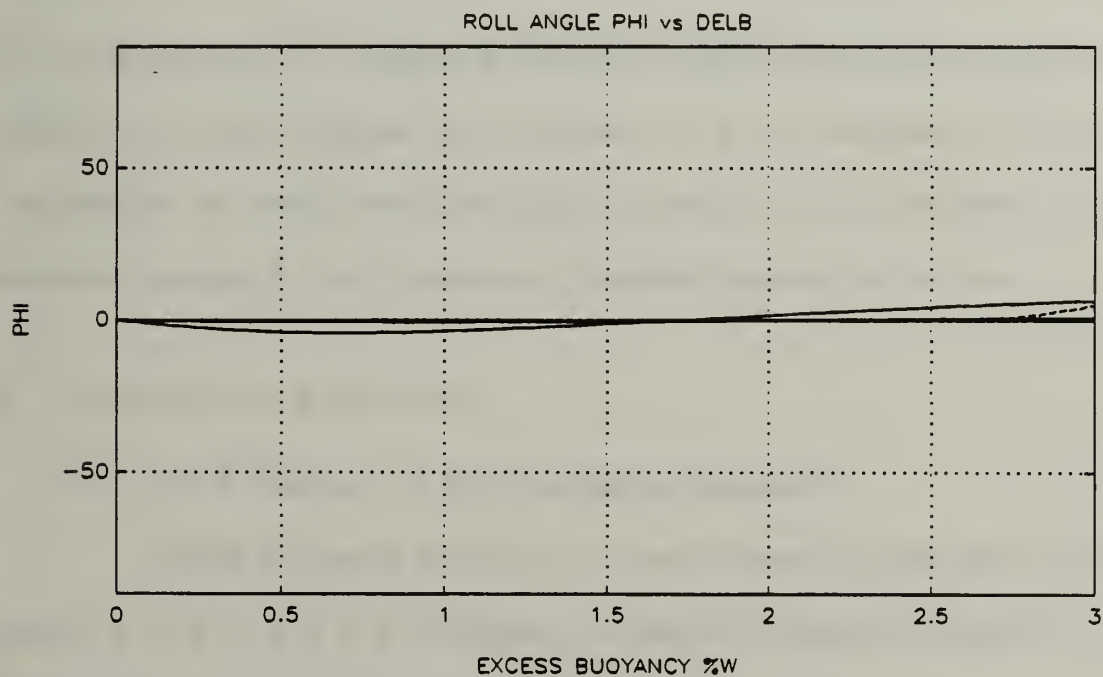


Figure 80. Steady State Roll Angle

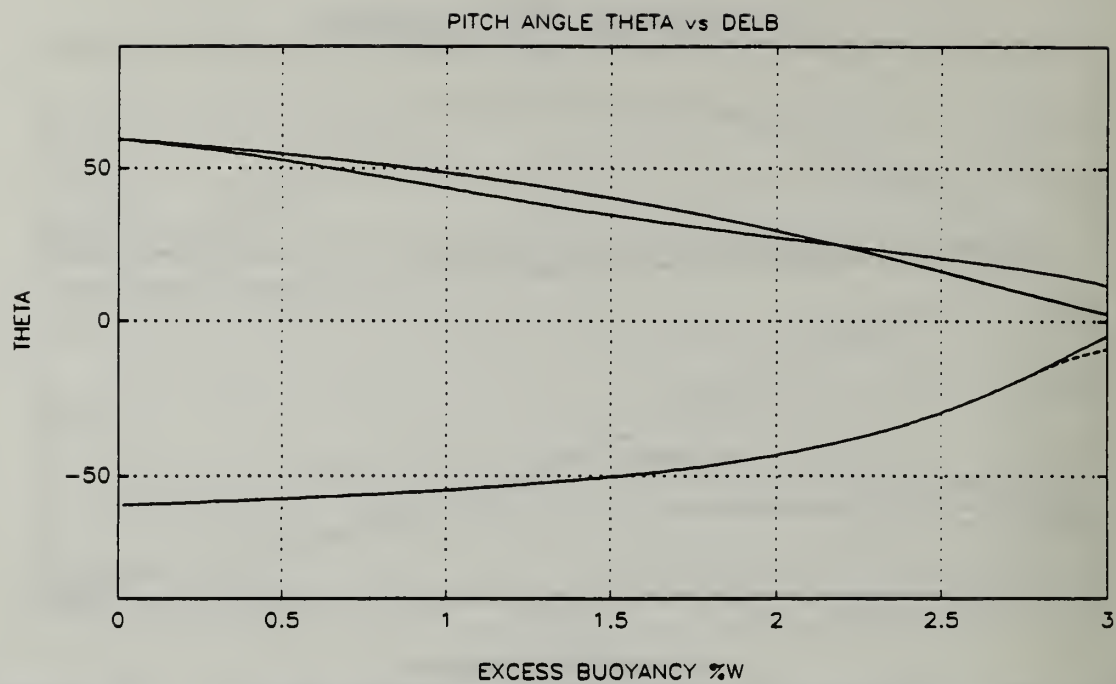


Figure 81. Steady State Pitch Angle

VII. CONTINUATION IN z_{GB}

A. GENERAL

In this chapter, the results of continuation in terms of the vertical center of buoyancy will be discussed. In continuation runs, the system of equations (35) was solved for changing values of the primary parameter λ (in this case $\lambda = z_{GB}$). In these runs, first we fixed the value of rudder angle and changed the values of sternplane angle, then we kept the sternplane angle fixed at zero and changed the rudder angle to see how the steady state values were changing. When we kept the rudder angle at zero and changed the values of sternplane angle, we saw that there was a pitchfork bifurcation for the state variables $u, v, w, p, r, \Phi, \theta$. After we saw that there was a pitchfork bifurcation when $\delta s = -20$ degrees and $\delta r = 0$ degrees, we started to change rudder angle δr to see how the pitchfork bifurcation diagrams were changing. As a last continuation run for this configuration, the value of stern plane angle was fixed at zero and the value of rudder angle was changed. We didn't observe any branching behavior for this case.

B. STEADY STATE RESULTS

1. $\delta r = 0$ Degrees, δs is the Perturbation Parameter

Figures 82 through 89 show steady state solutions for eight state variables, namely; $u, v, w, p, q, r, \Phi, \theta$. In Figures 82 through 89, solid lines correspond to $\delta s = 0$ degrees, dashed lines correspond to $\delta s = -5$ degrees and dash-dot lines correspond to

$\delta s = -20$ degrees. In all of these cases, the vertical center of buoyancy is the primary continuation parameter, and δs is the perturbation parameter, while δr is kept at zero degrees. Values of the other parameters are kept fixed: Excess buoyancy $\delta B = 2\%$ of the vehicle weight (W); longitudinal center of buoyancy $x_{GB} = -1\%$ of vehicle length; deflection of the bow planes $\delta b = 0$; location of the vertical and horizontal centers of buoyancy, $z_B = x_B = 0$. As we see from Figures 82 through 89, there is a pitchfork bifurcation for state variables $u, v, w, p, r, \Phi, \theta$. We have out of plane solutions for this vehicle configuration.

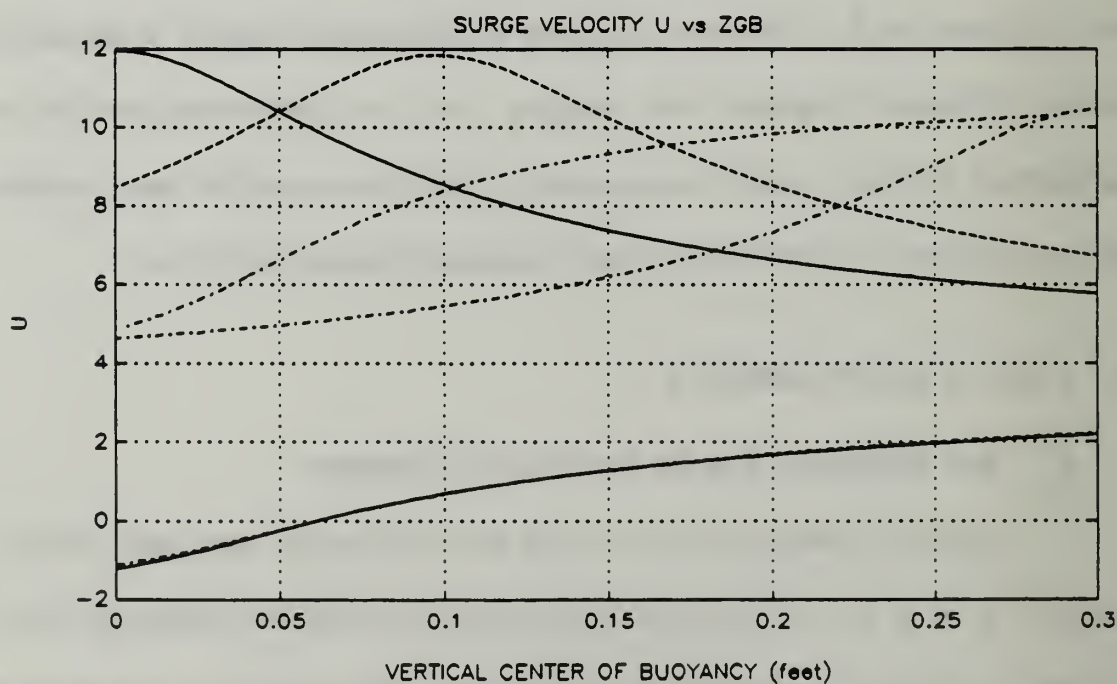


Figure 82. Steady State Surge Velocity

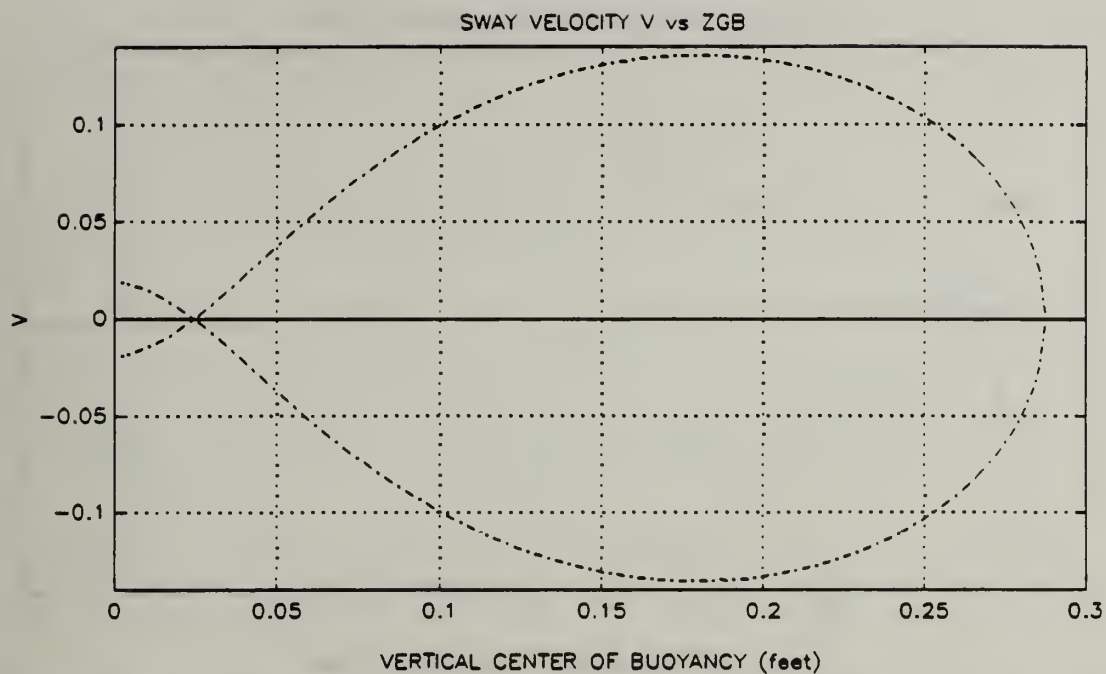


Figure 83. Steady State Sway Velocity

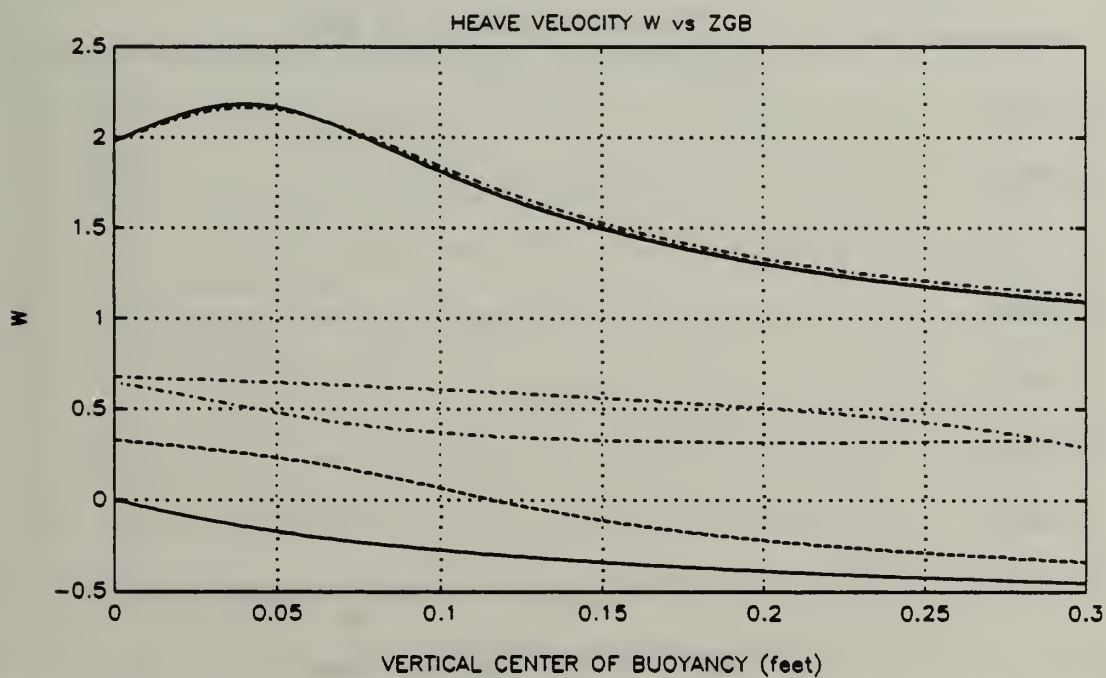


Figure 84. Steady State Heave Velocity

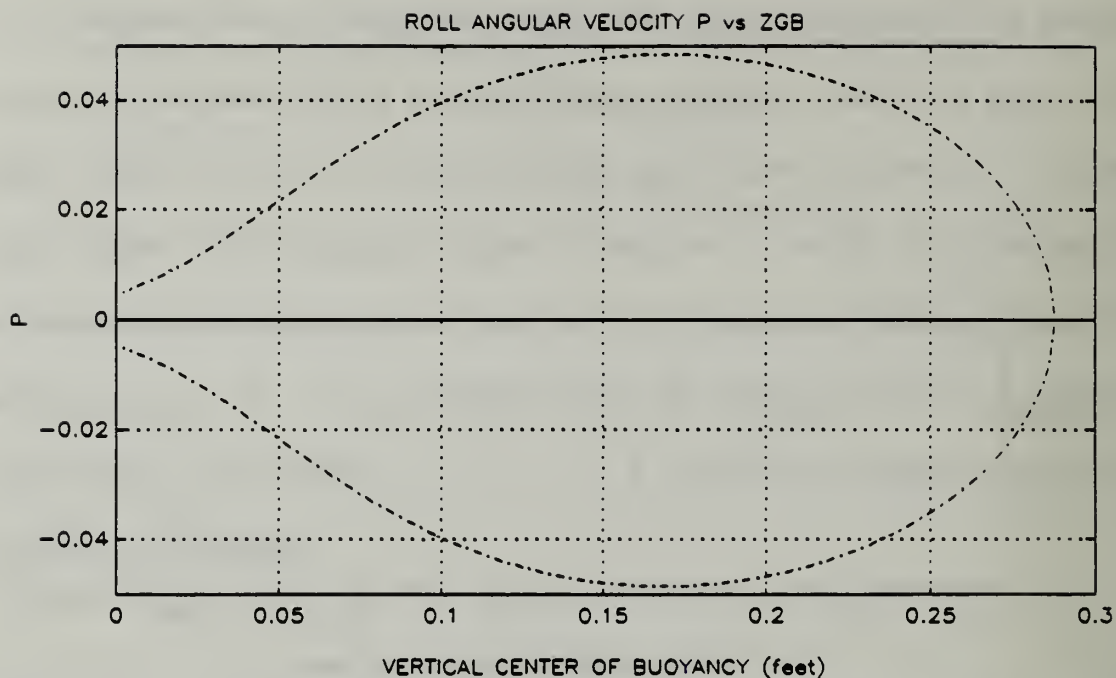


Figure 85. Steady State Roll Angular Velocity

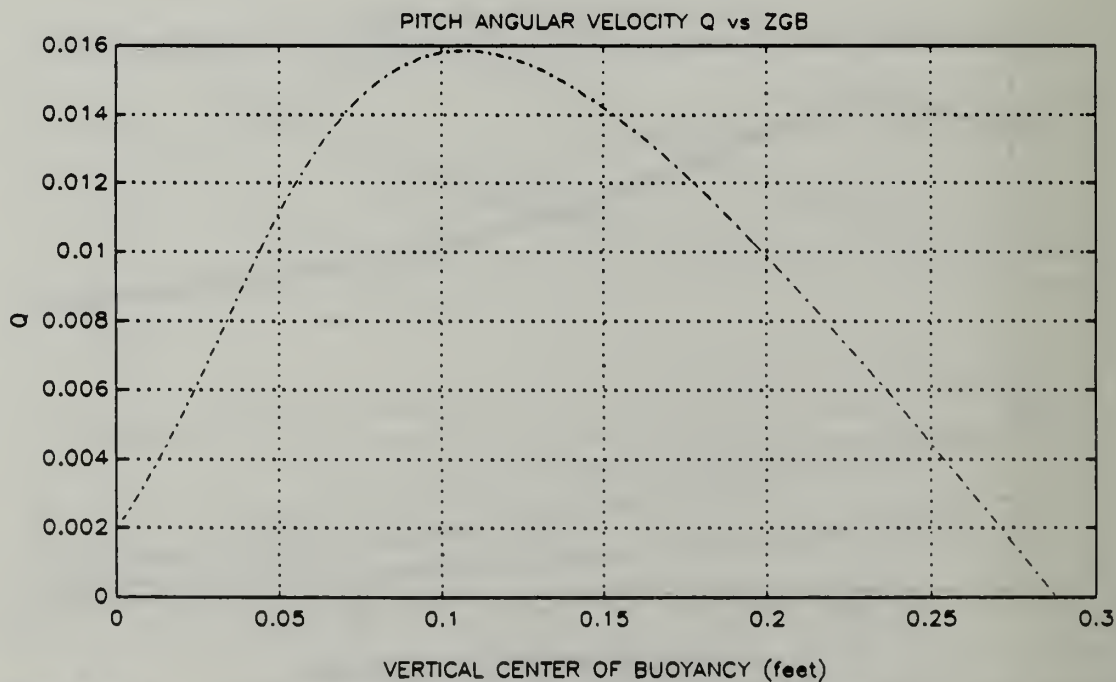


Figure 86. Steady State Pitch Angular Velocity

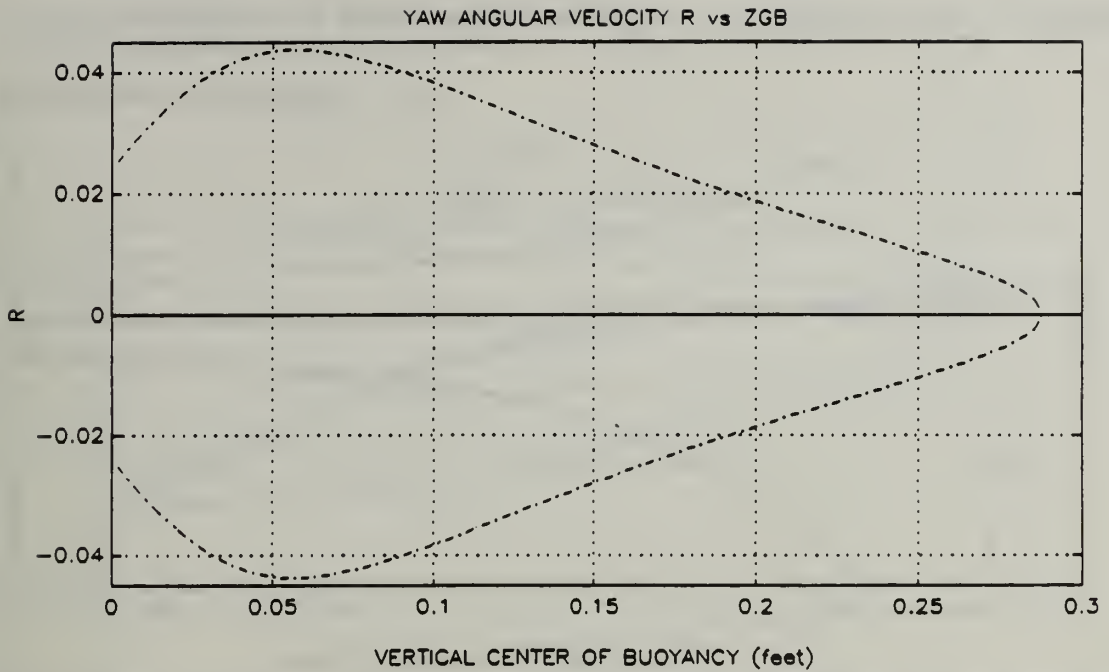


Figure 87. Steady State Pitch Angular Velocity

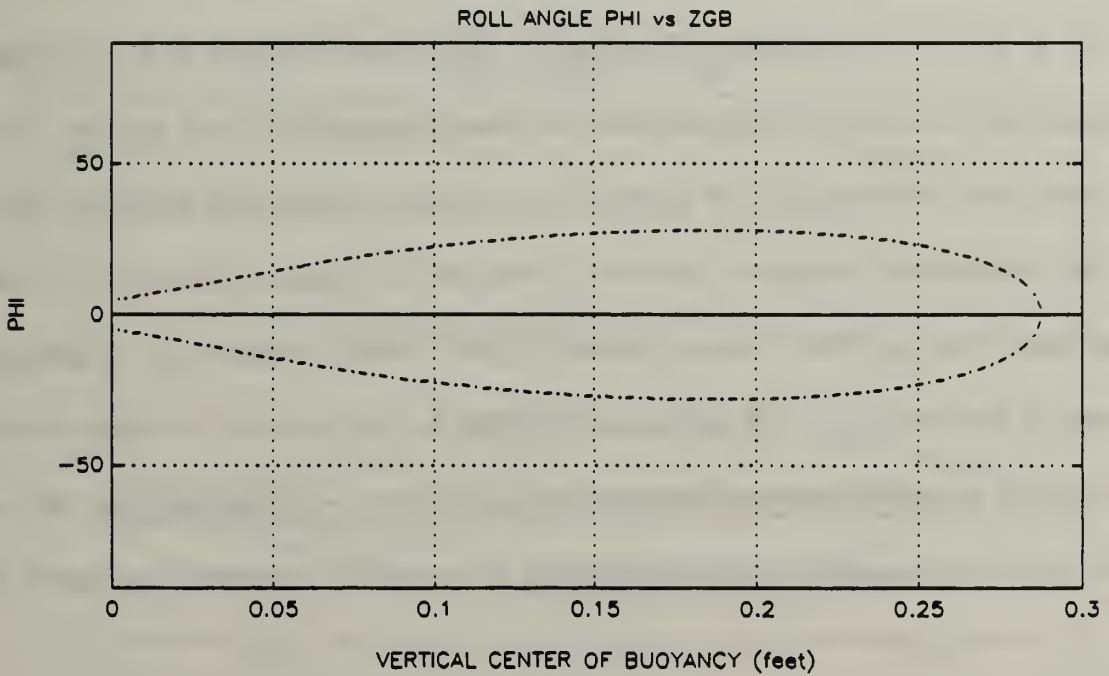


Figure 88. Steady State Roll Angle

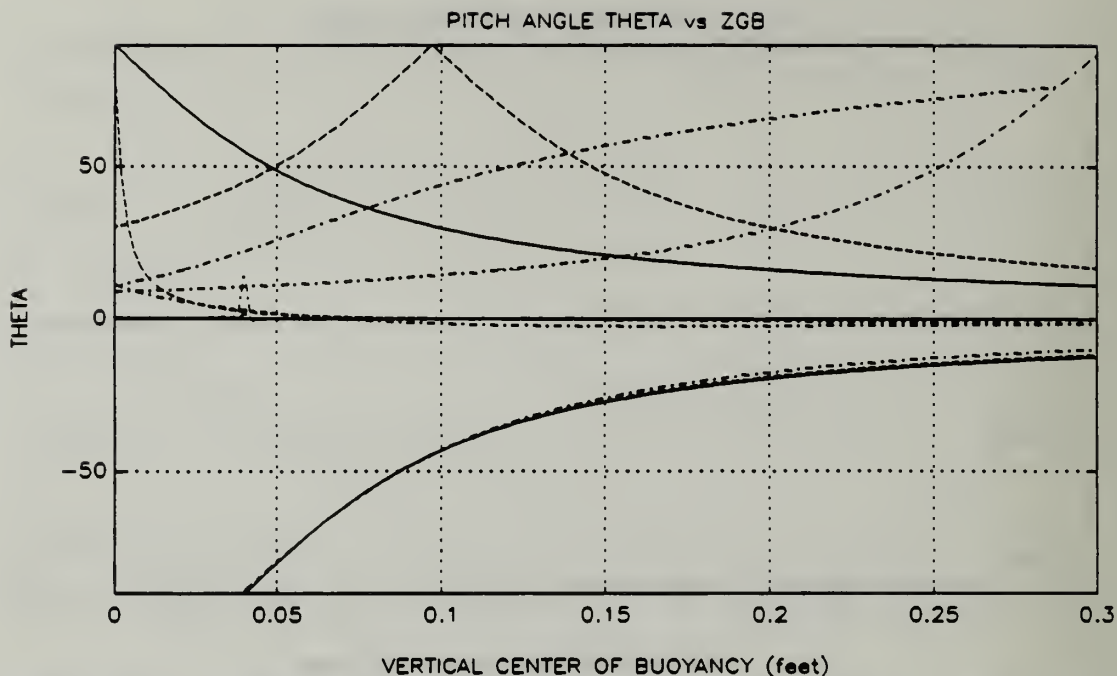


Figure 89. Steady State Pitch Angle

2. $\delta s = -20$ Degrees, δr is the Perturbation Parameter

Figures 90 through 97 show steady state solutions for the state variables u , v , w , p , q , r , Φ , θ . In Figures 90 through 97, solid lines correspond to $\delta r = 0$ degrees, dashed lines when $\delta r = 0.5$ degrees, dash-dot lines correspond to $\delta r = 5$ degrees. In all of these cases, vertical center of buoyancy z_{GB} is primary continuation parameter and δr is the perturbation parameter, while δs is kept at -20 degrees. Values of the other parameters are kept fixed; excess buoyancy $\delta_b = 2\%$ vehicle weight (W), longitudinal center of buoyancy $x_{GB} = -1\%$ of the vehicle length (L); deflection of bowplanes $\delta b = 0$; location of horizontal and vertical centers of buoyancy, $x_B = z_B = 0$. By doing this, we saw how the bifurcation points were changing as we change perturbation parameter δr .

A bifurcation phenomenon took place for $\delta r = 0$ degrees and disappeared after we applied a small nonzero rudder angle value.

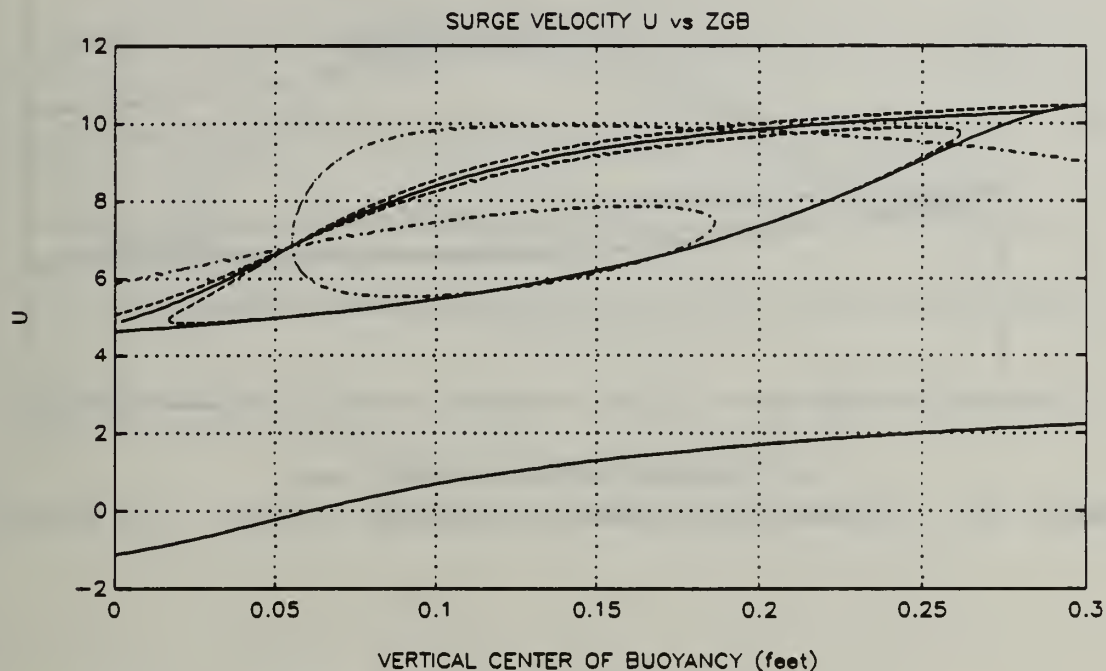


Figure 90. Steady State Surge Velocity

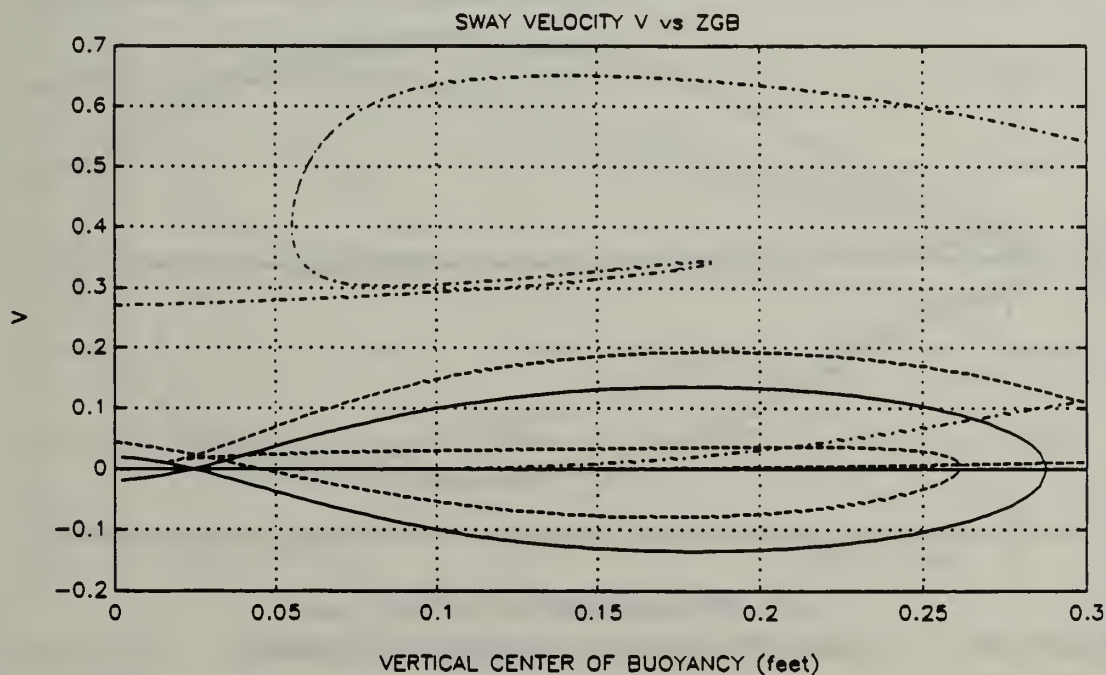


Figure 91. Steady State Sway Velocity

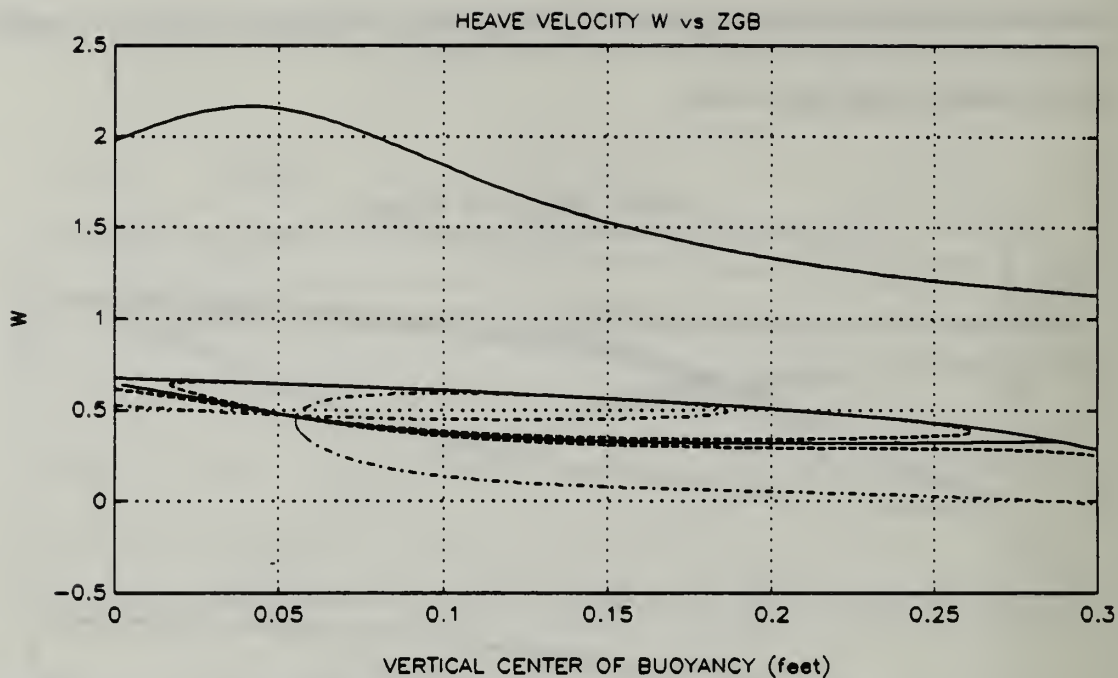


Figure 92. Steady State Heave Velocity

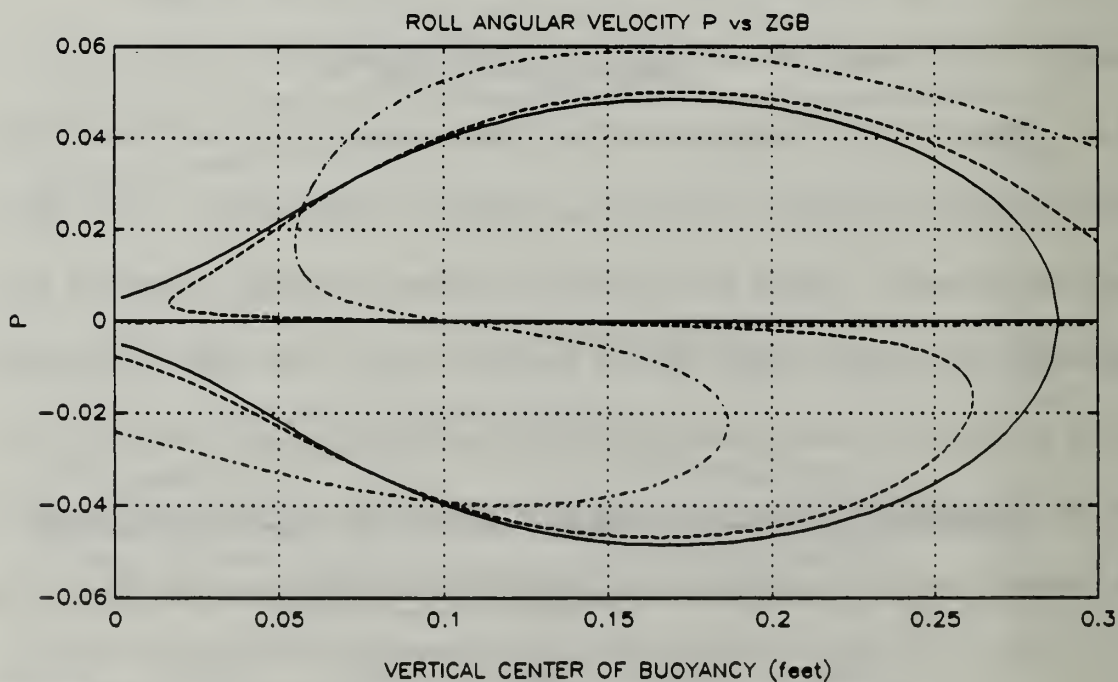


Figure 93. Steady State Roll Angular Velocity

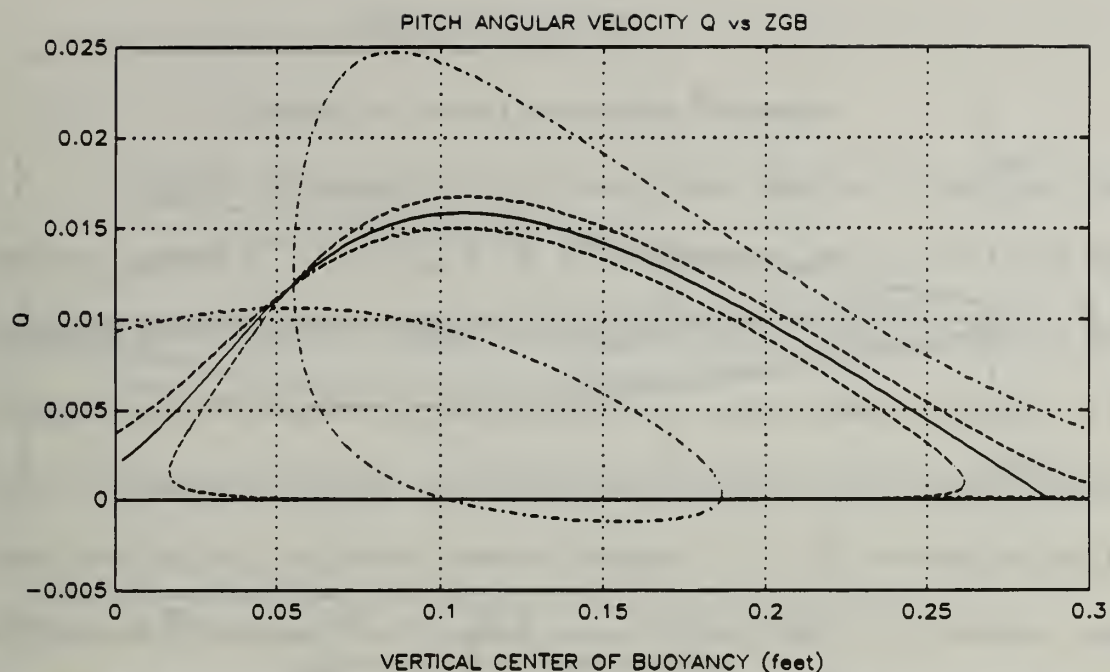


Figure 94. Steady State Pitch Angular Velocity

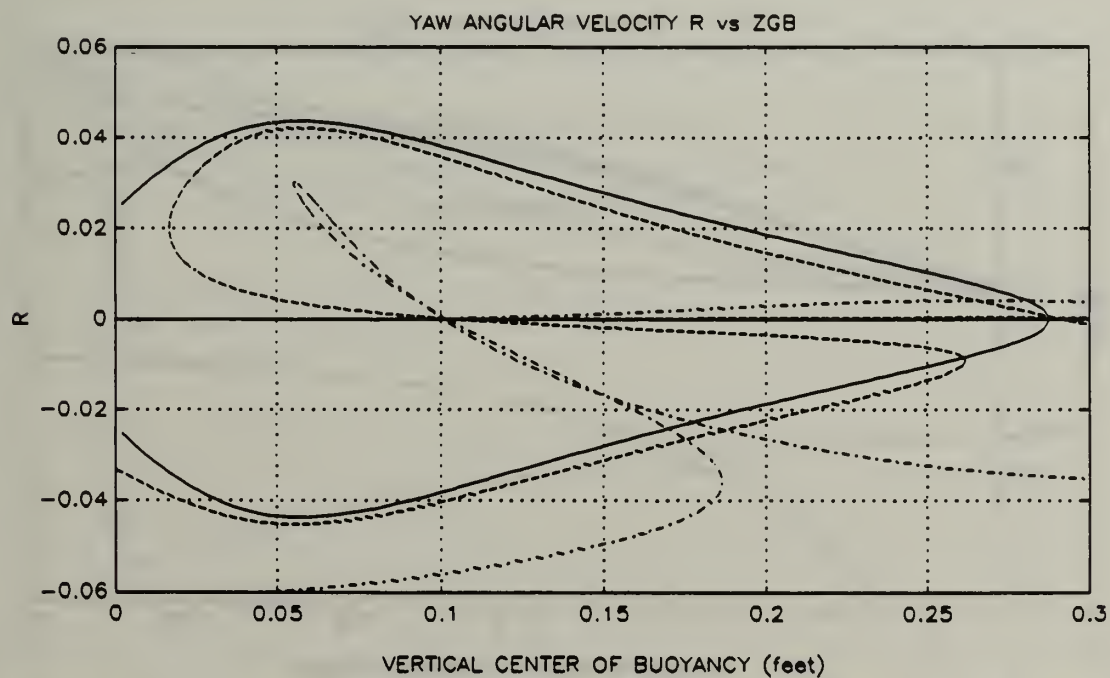


Figure 95. Steady State Yaw Angular Velocity

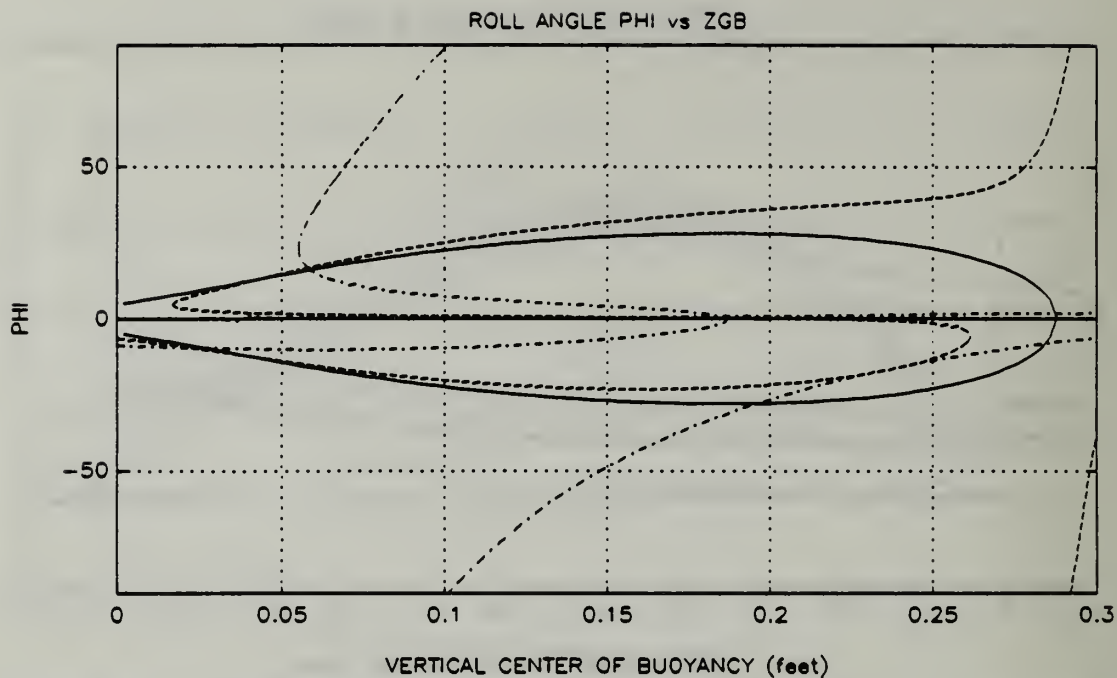


Figure 96. Steady State Roll Angle

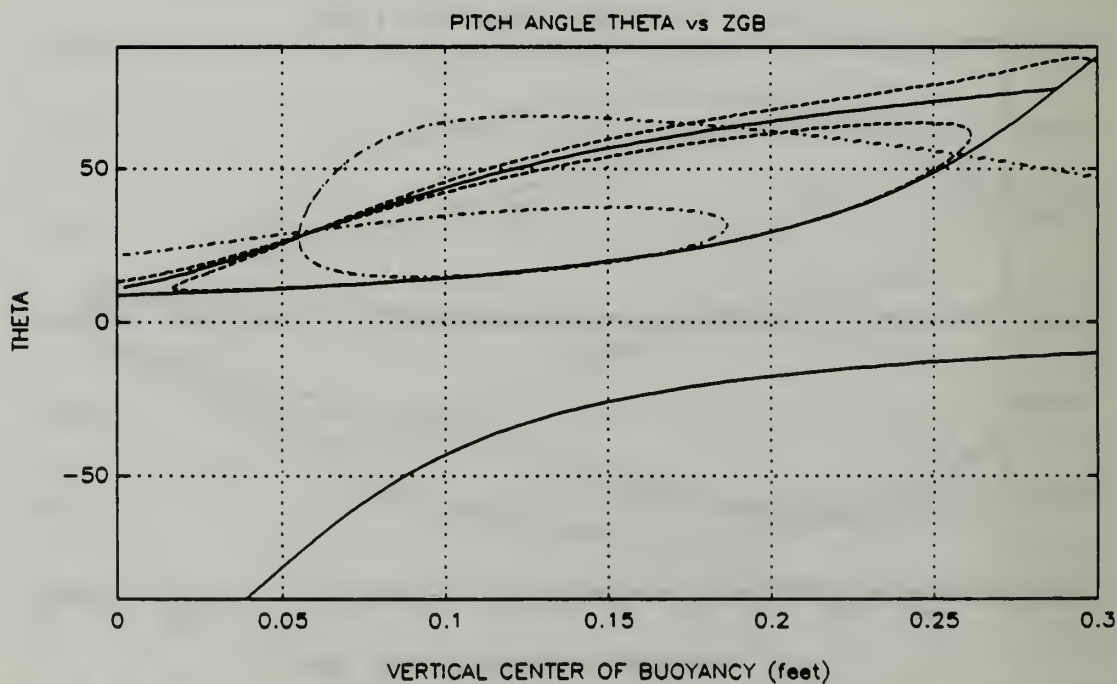


Figure 97. Steady State Pitch Angle

3. $\delta s = 0$ Degrees, δr is the Perturbation Parameter

Figures 98 through 105 show steady state solutions for the eight state variables, namely u , v , w , p , q , r , Φ , θ . In Figures 98 through 105, solid lines correspond to $\delta r = 0$ degrees, dashed lines correspond to $\delta r = .5$ degrees, dash-dot lines correspond to $\delta r = 5$ degrees. In all these cases, the vertical center of buoyancy is the primary continuation parameter. Values of the other parameters are kept fixed; sternplane action $\delta s = 0$ degrees; longitudinal center of buoyancy $x_{GB} = -1\%$ of vehicle length (L); deflection of mow planes, $\delta b = 0$ degrees; excess buoyancy $\delta B = 2\%$ of vehicle weight (W); location of horizontal and vertical centers of buoyancy $x_B = z_B = 0$. As we see, there

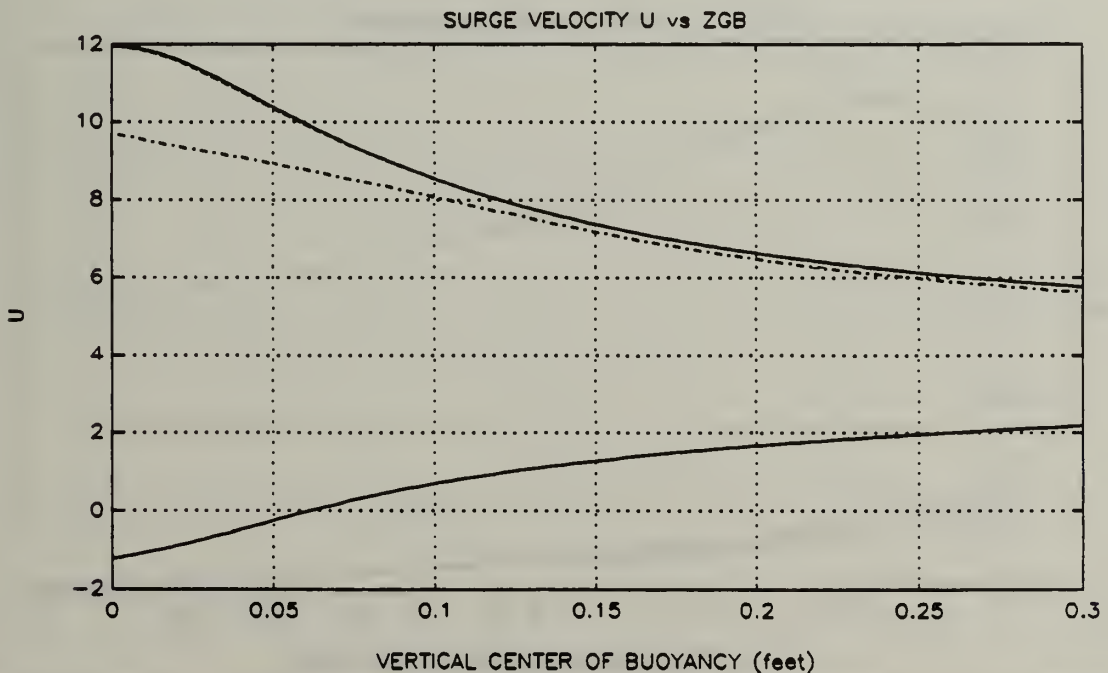


Figure 98. Steady State Surge Velocity

is no solution branching for this last case. We have out of plane solutions for nonzero rudder angles.

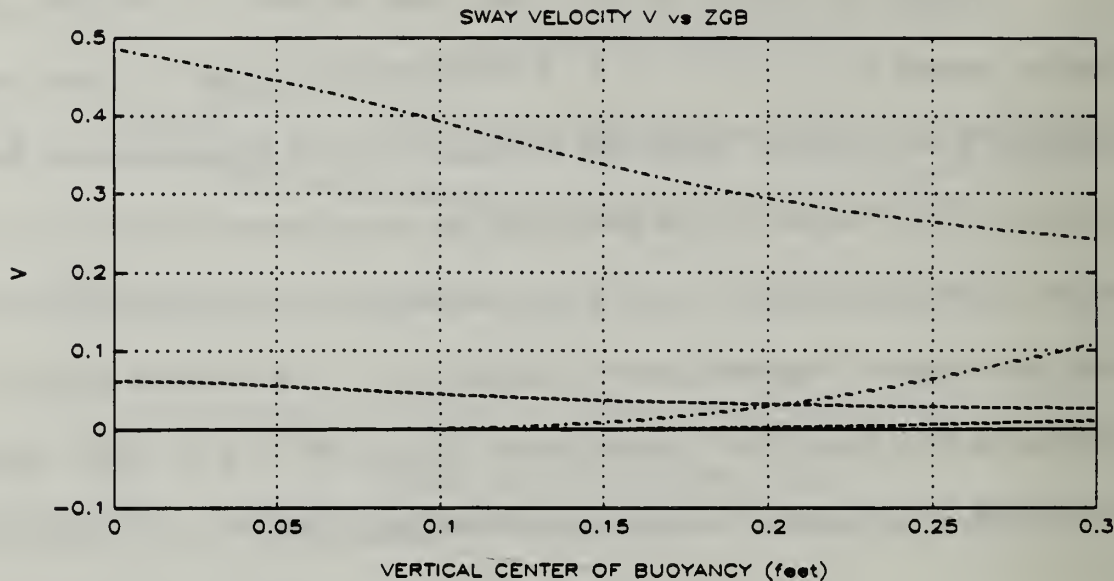


Figure 99. Steady State Sway Velocity

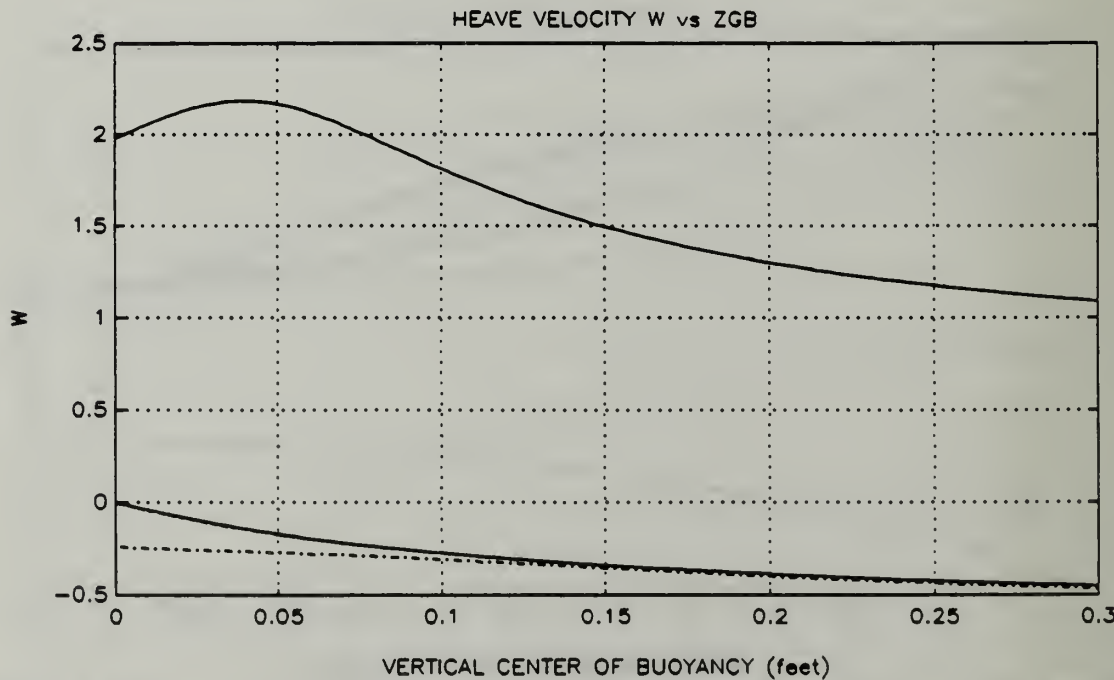


Figure 100. Steady State Heave Velocity

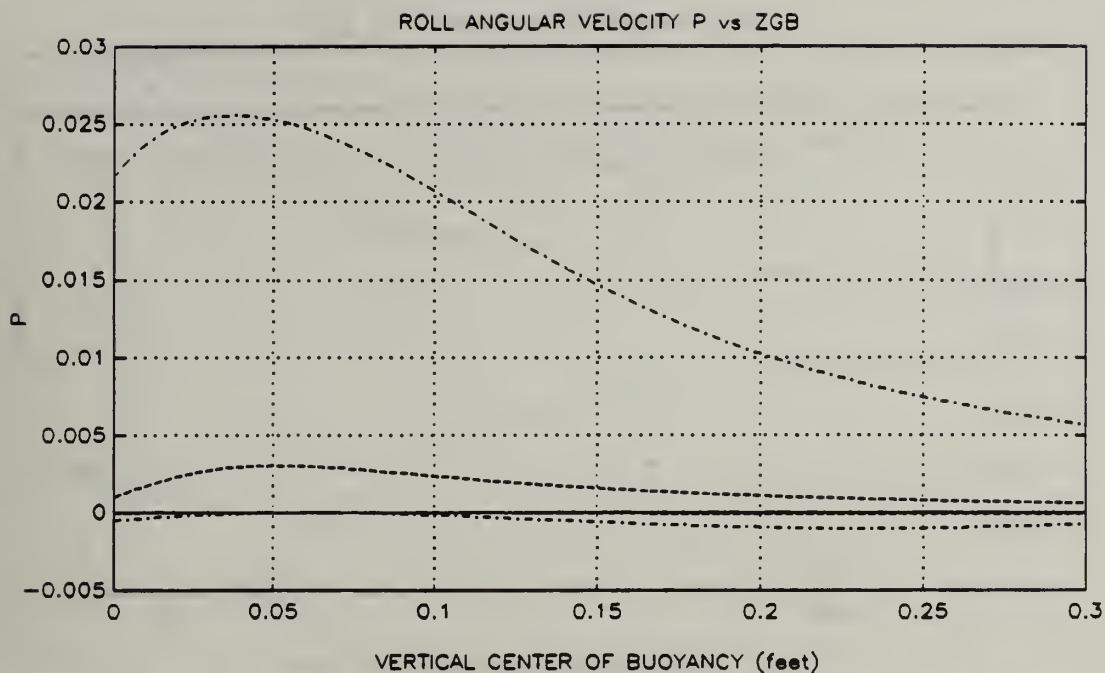


Figure 101. Steady State Roll Angular Velocity

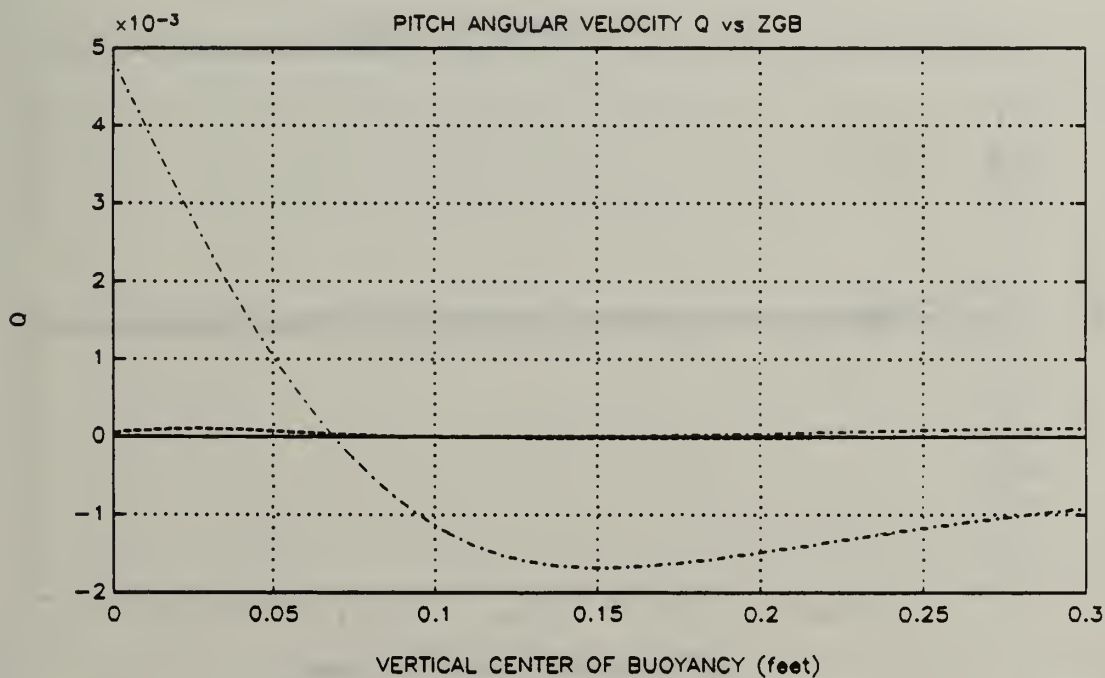


Figure 102. Steady State Pitch Angular Velocity

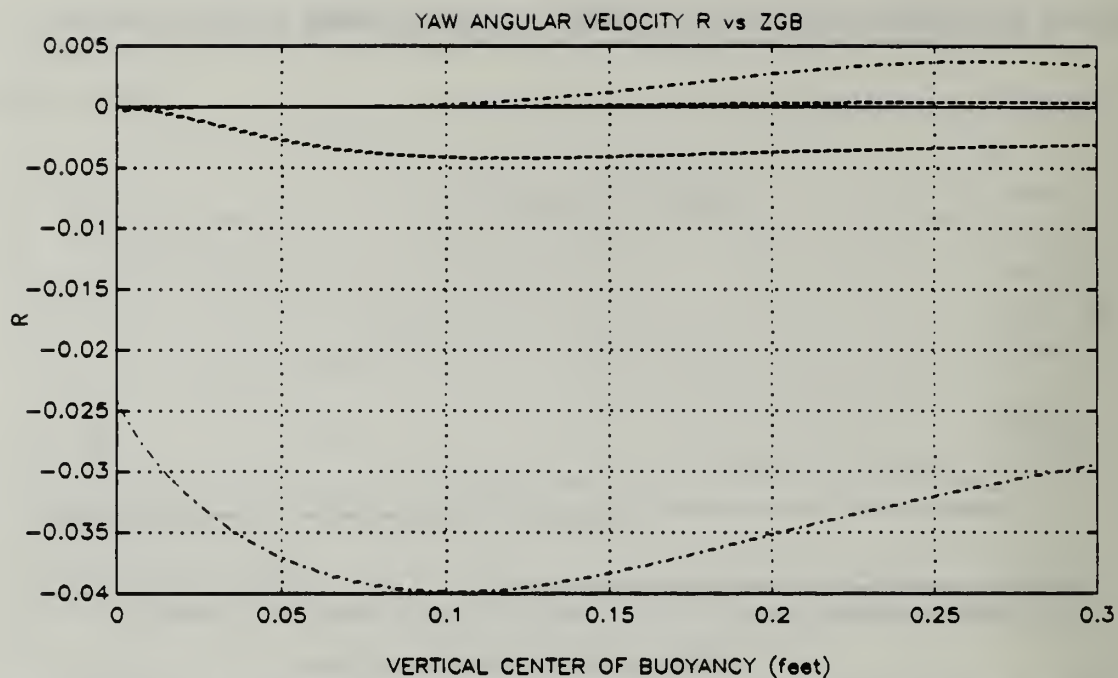


Figure 103. Steady State Yaw Angular Velocity

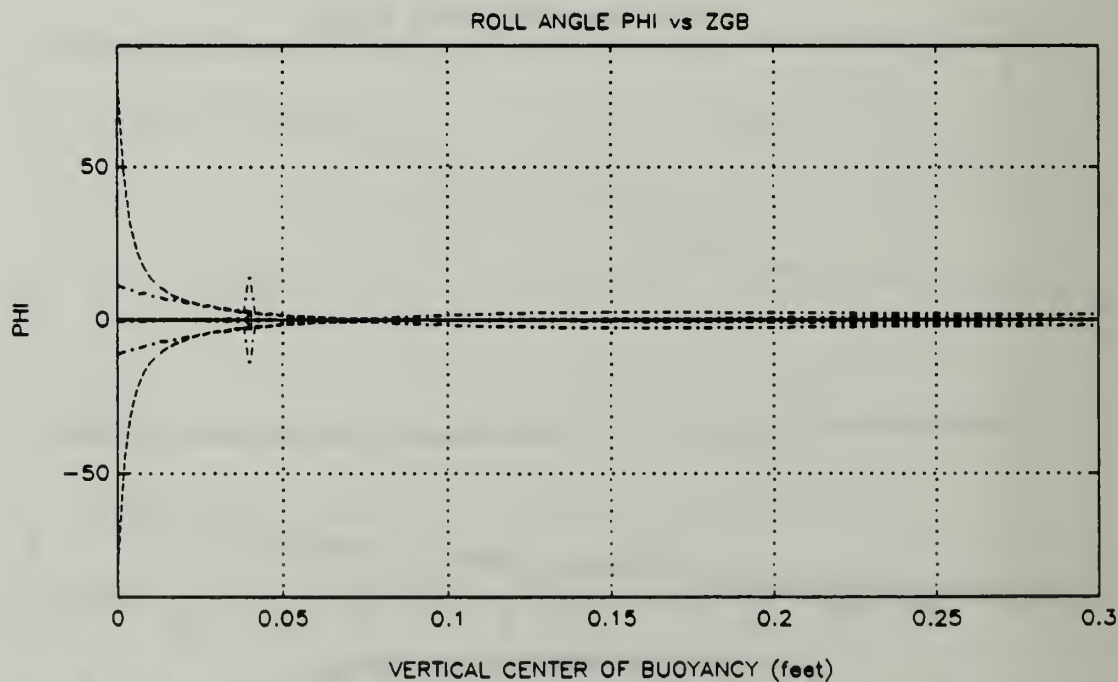


Figure 104. Steady State Roll Angle

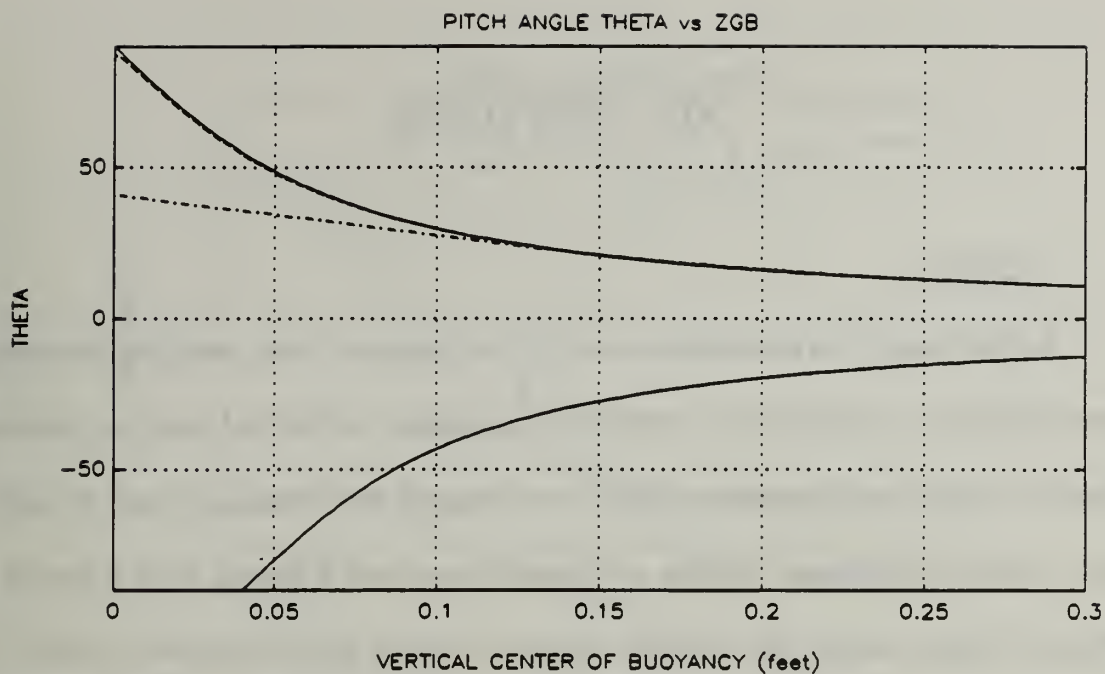


Figure 105. Steady State Pitch Angle

VIII. CUSP CURVES

A. GENERAL

In this chapter, the cusp phenomenon will be discussed. After seeing the branching characteristics of the steady state solutions of our system, we decided to look at how the bifurcation points and the turning points were changing with changing values of rudder angle. The main difference between a bifurcation point and a turning point is that, at a bifurcation point, exactly two branches intersect with two distinct tangents; whereas, at a turning point, the branch comes from one side and turns back. We can see this phenomenon from Figure 106. This is the graph that was generated for the surge velocity when the primary continuation parameter was x_{CB} and the sternplane angle $\delta s = -20$ degrees. In Figure 106, points A and B are bifurcation points, and points C and D are turning points. Clearly, locally there are no solutions on one side of a turning point and two solutions on the other side, but there are solutions on each side of a bifurcation point.

Let us look at Equation 43 again, this is the model equation that represents a cusp,

$$x^3 - Bx + A = 0 \quad (43)$$

Let us call this algebraic equation as a function of x , $F(x) = 0$. For the critical curve, F and dF/dx should be zero.

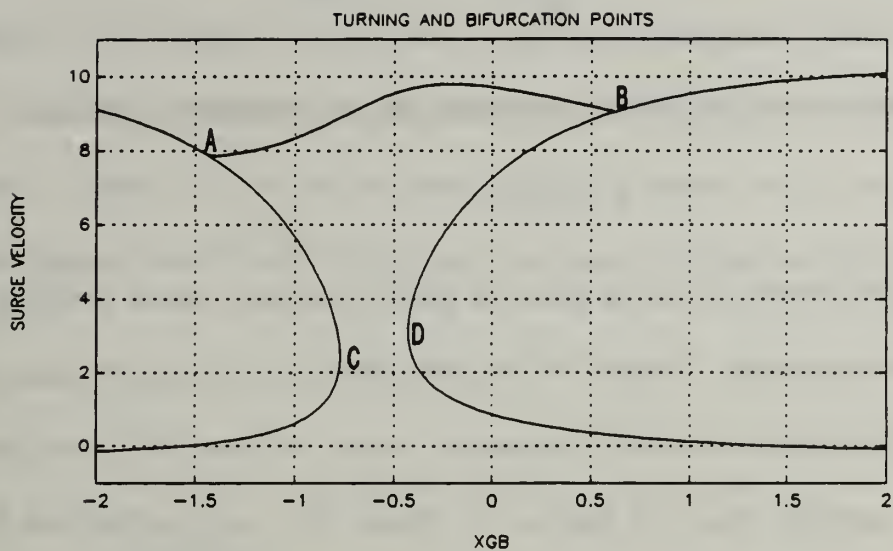


Figure 106. Turning and Bifurcation Points

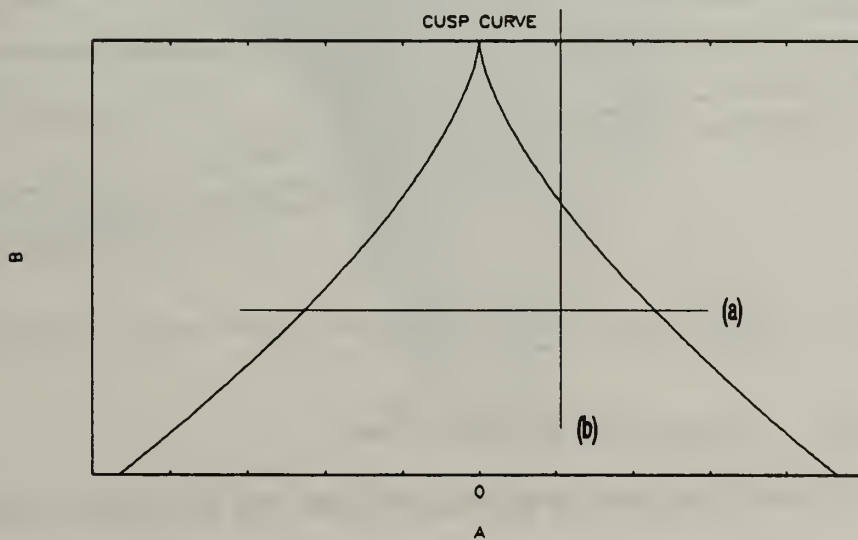


Figure 107. Cusp Curve

$$\frac{dF}{dx}=0 \quad \Rightarrow \quad x^2 = \frac{B}{3}$$

If we substitute this value into equation (43), we get the following relation:

$$4B^3 = 27A^2$$

If we plot this relation in the AB plane, we get the cusp curve shown in Figure 107. This represents a separation. Equation 43 has three solutions for parameter values inside the cusp and one solution outside this parameter values. In Figure 107, if we fix the value of A and vary the value of B (path (a) in Figure 107) we get a pitchfork bifurcation diagram, as shown in Figure 108.

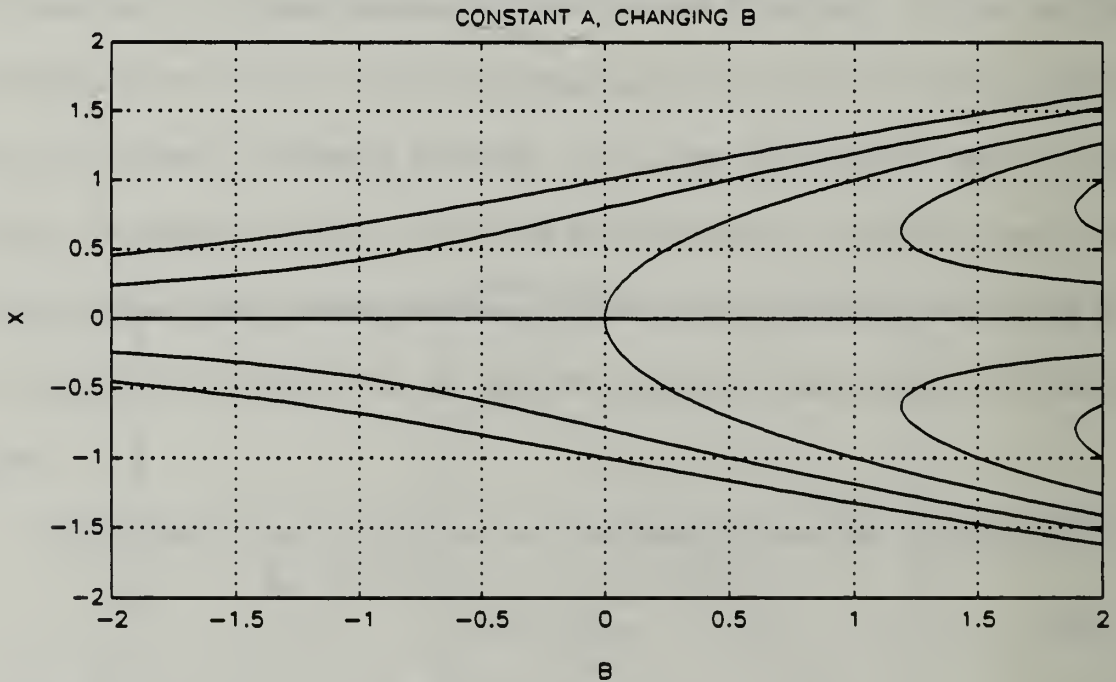


Figure 108. Pitchfork Bifurcation

In Figure 107, if we fix the value of the value of B and vary the value of A (path (b) in Figure 107) we get a hysteresis behavior, as shown in Figure 110. It is clear, that when

a line crosses the cusp curve in the (A, B) plane a branching phenomenon takes place. As mentioned before, Equation 43 has three solutions for parameter values inside the cusp and one solution outside the cusp this can be seen easily in Figure 109. Now let us take a look at Figure 111. As can be seen, there is a simple way of recovering a bifurcation diagram from the path through the given cusp by lifting the path to the cusp surface.

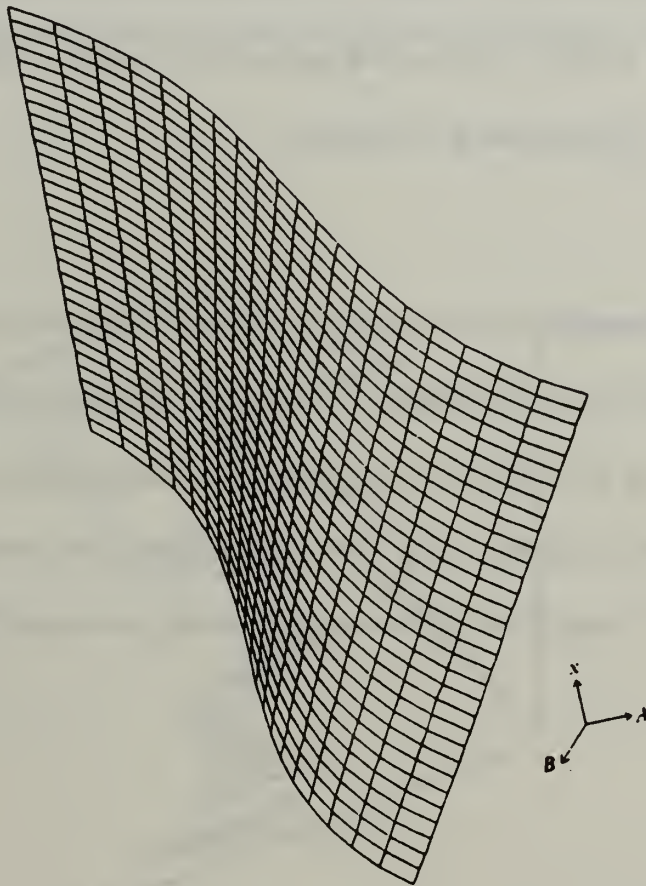


Figure 109. 3-D Graph of Equation 43

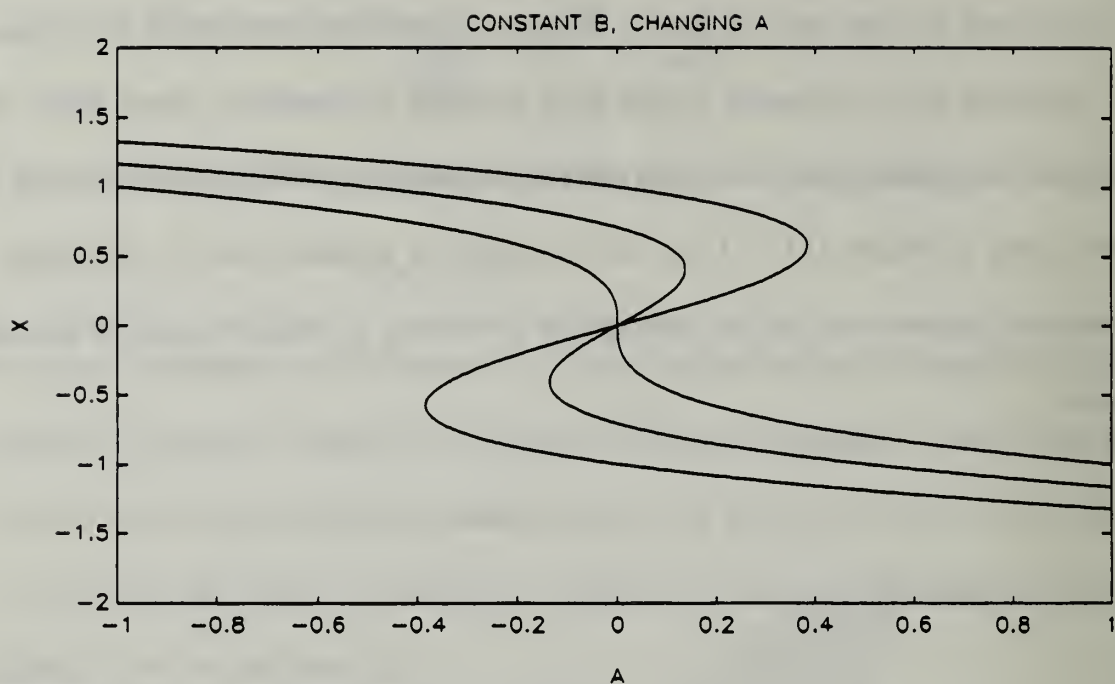


Figure 110. Hysteresis Diagram

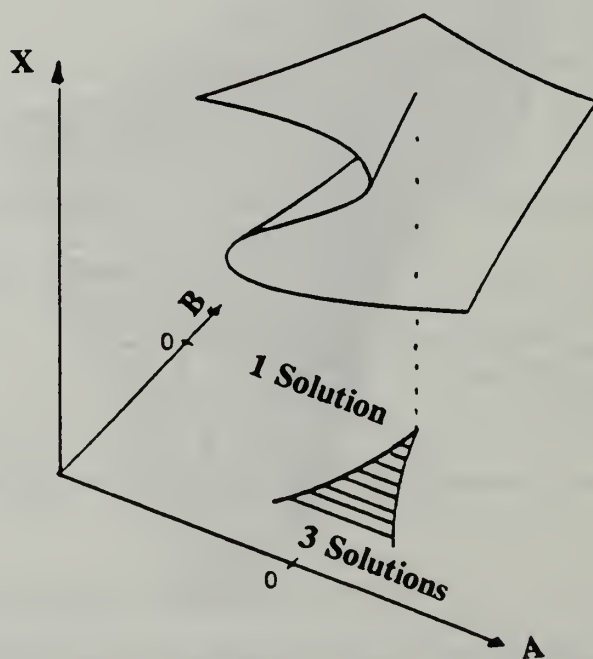


Figure 111. Geometry of Cusp Phenomenon

If we have the cusp curves for a multiparameter equations system, we can always have an idea about the solutions of this system for different perturbed cases. This means that by looking at the path of the parameters through the cusp curve, we can see how many solutions we have for the corresponding configuration of the system. Similarly, we can think about another problem which has a turning point. If we draw the position of turning point in the plane with changing parameter values, we will get two solutions inside the curve and no solutions outside these parameter values. Therefore, these graphs can tell us the nature of the solution set of our system. In our study, we tried to get the cusp curves and the position of the turning points and the results are presented in the following section.

B. RESULTS

First of all, let us look at the case when we have the sternplane action as our primary continuation parameter. When δs is between -5 and -10 degrees, we had a bifurcation point and in that case the rudder angle was zero, see Figure 13. To get the complete cusp curve, we changed the value of δr and found the locations of the bifurcation point. These were plotted as δs versus δr , see Figure 112.

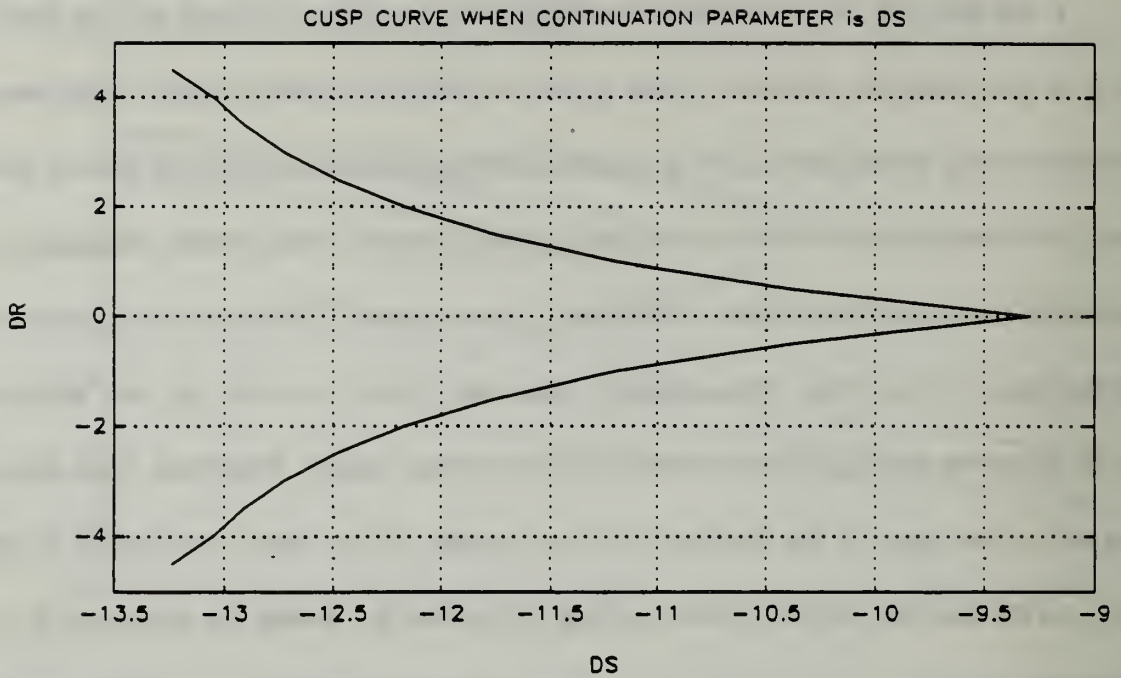


Figure 112. Cusp Curve When Continuation Parameter is Sternplane angle

In Chapter V, we saw that when we have x_{GB} as primary continuation parameter, and sternplane action $\delta s = -20$ degrees, we have both bifurcation and turning points, Figure 106. To see the effect of changing the rudder angle on the bifurcation and turning points, we changed the value of rudder angle and found the location of bifurcation and turning points in the $(x_{GB}, \delta r)$ plane. The results are shown on the following graphs. The bifurcation points generated a cusp curve and the turning points generated a parabola in the $(x_{BG}, \delta r)$ plane.

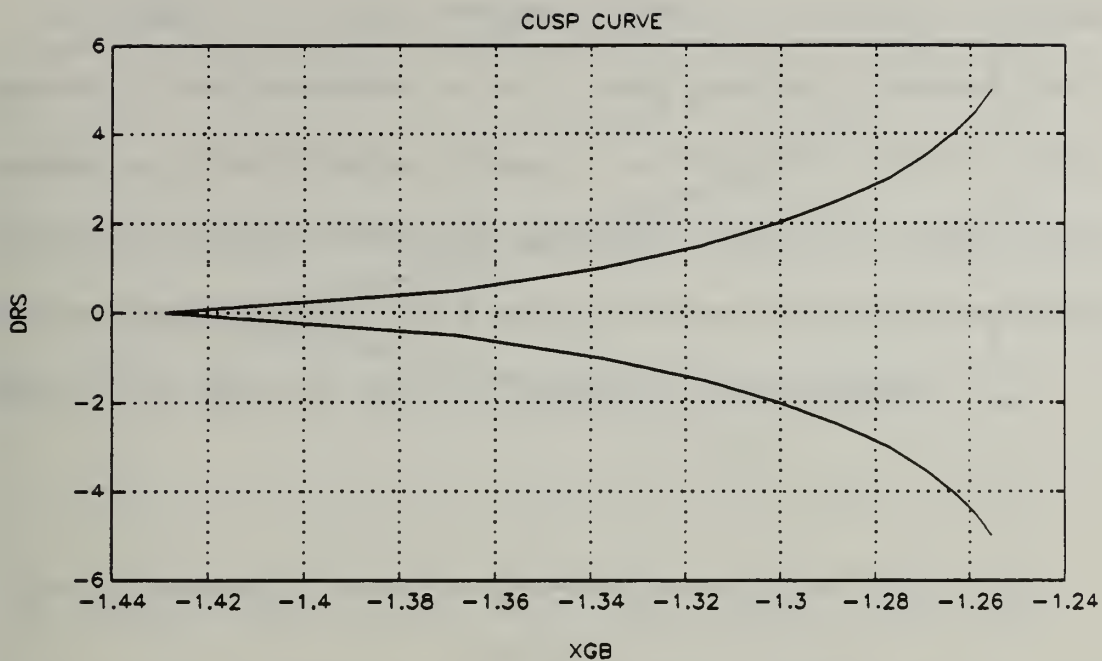


Figure 113. Cusp Curve When Continuation Parameter is x_{GB}

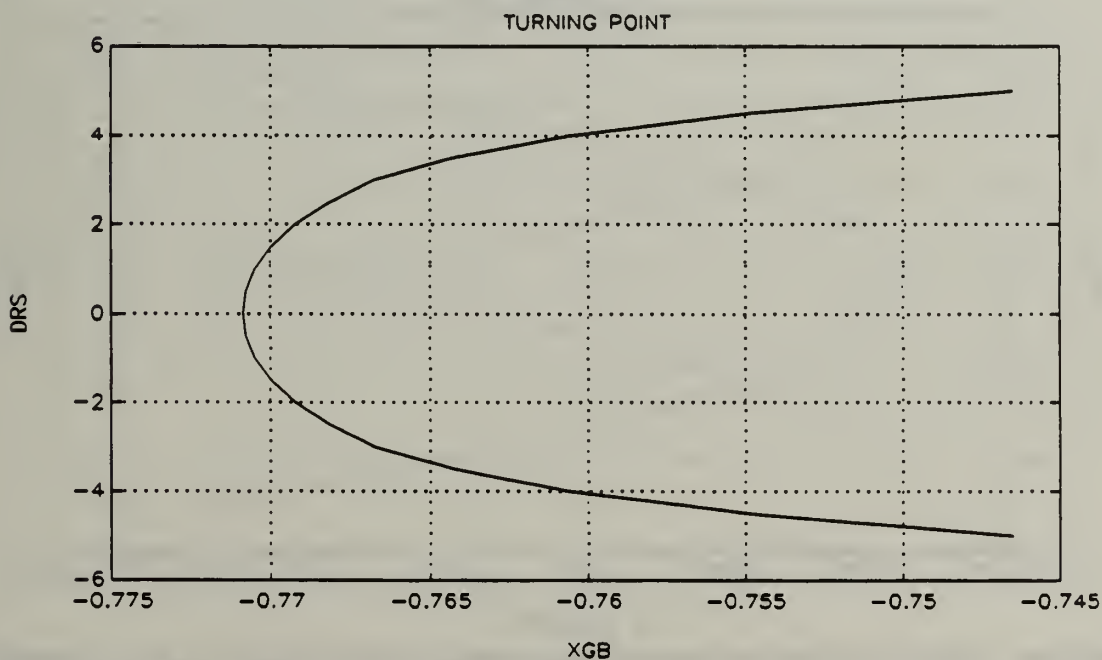


Figure 114. Turning Point When Continuation Parameter is x_{GB}

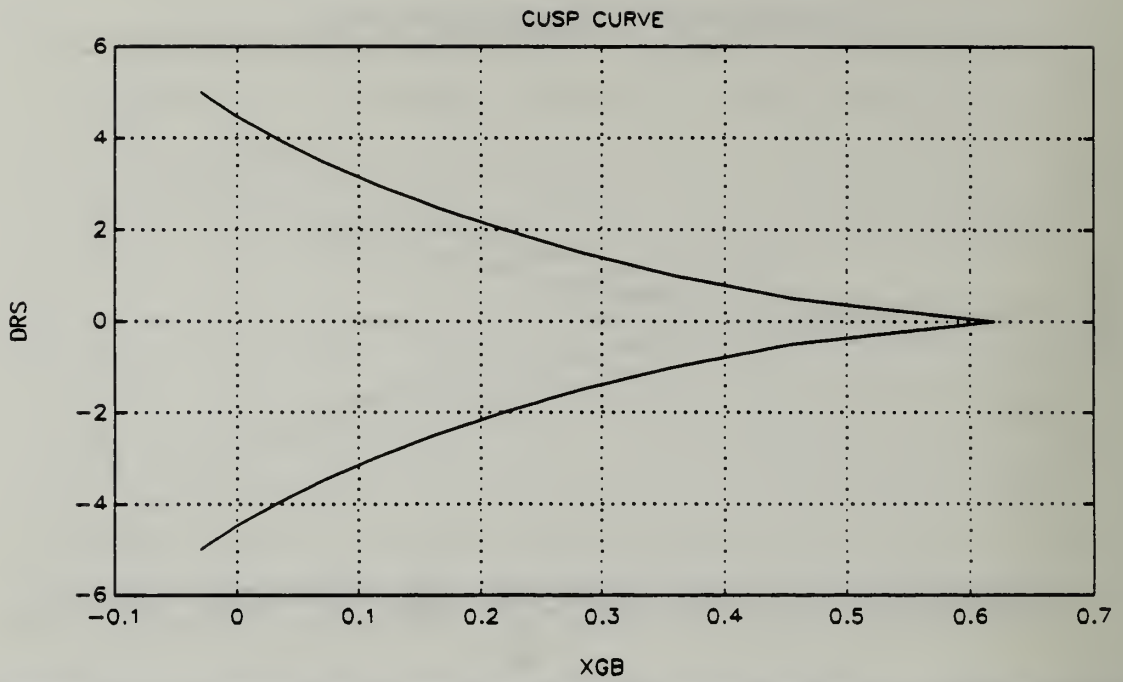


Figure 115. Second Cusp Curve When Continuation Parameter is x_{GB}

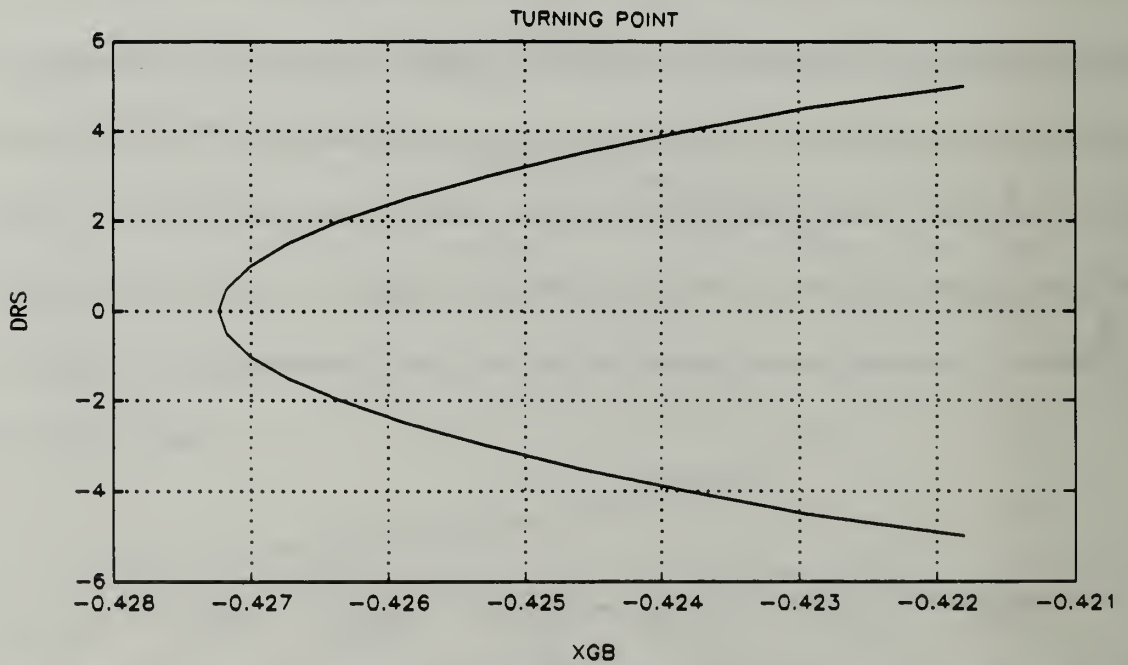


Figure 116. Second Turning Point When Continuation Parameter is x_{GB}

In Chapter VI, we used the excess buoyancy as our primary continuation parameter. In that configuration, we had a turning point and a bifurcation point for sternplane angle $\delta s = -20$ degrees, (Figure 57 in Chapter VI). To see the effect of different rudder angles on these points, we changed the value of the rudder angle and found the location of these points in the $(x_{GB}, \delta r)$ plane. These results are shown in Figure 117 and Figure 118. Again, we got a cusp curve and a parabola.

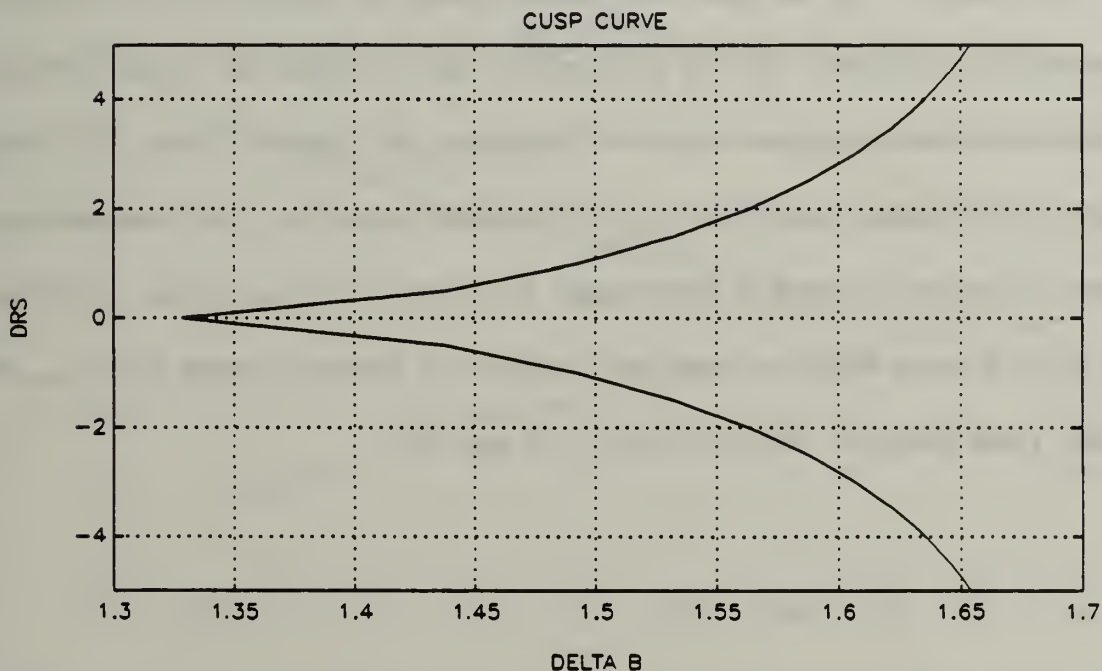


Figure 117. Cusp Curve When Continuation Parameter Is Excess Buoyancy

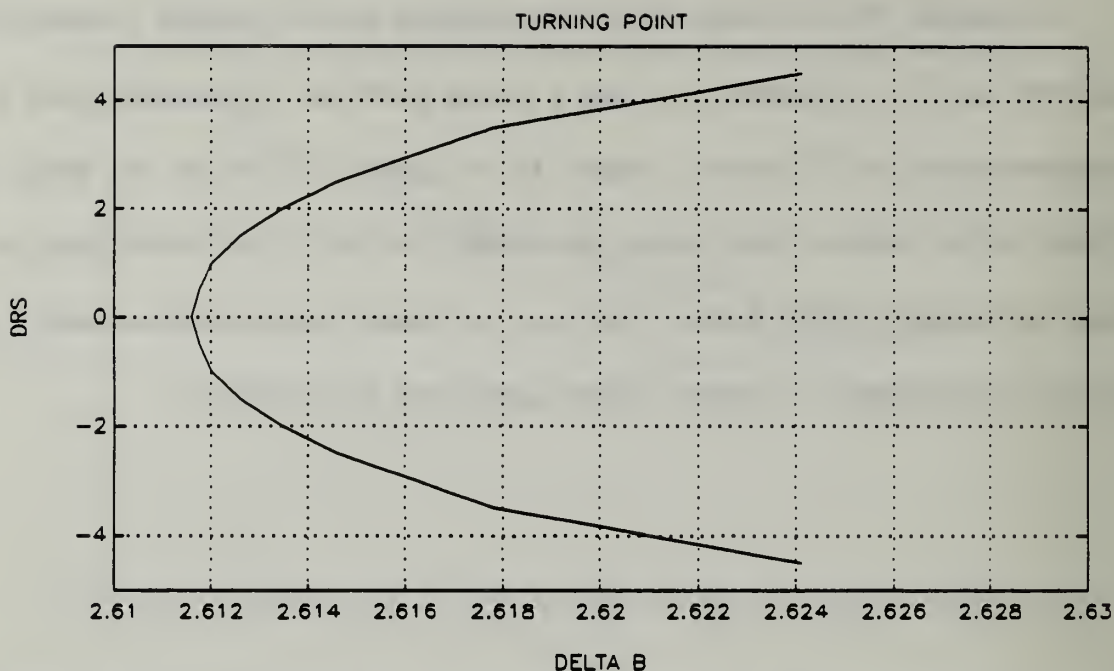


Figure 118. Turning Point When Continuation Parameter is Excess Buoyancy

In Chapter VII, we used the vertical center of buoyancy as our primary continuation parameter. In that continuation, run we found that there were two bifurcation points at sternplane angle $\delta s = -20$ degrees, see Figure 82 Chapter VII. In that figure, the bifurcation point when z_{GB} is around zero is not clear, but it becomes clear when we changed the value of rudder angle. To construct the cusp curves, we changed the value of rudder angle and found the coordinates of bifurcation points in the $(z_{GB}, \delta r)$ plane. These graphs are shown in Figures 119 and 120.

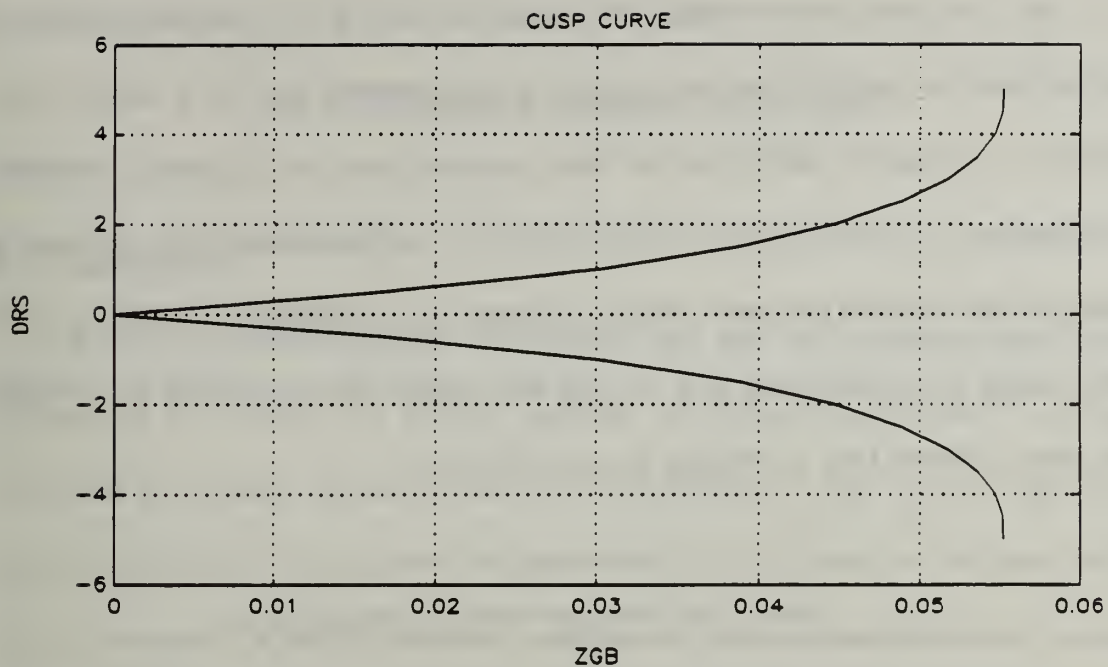


Figure 119. Cusp Curve When Continuation Parameter is z_{GB}

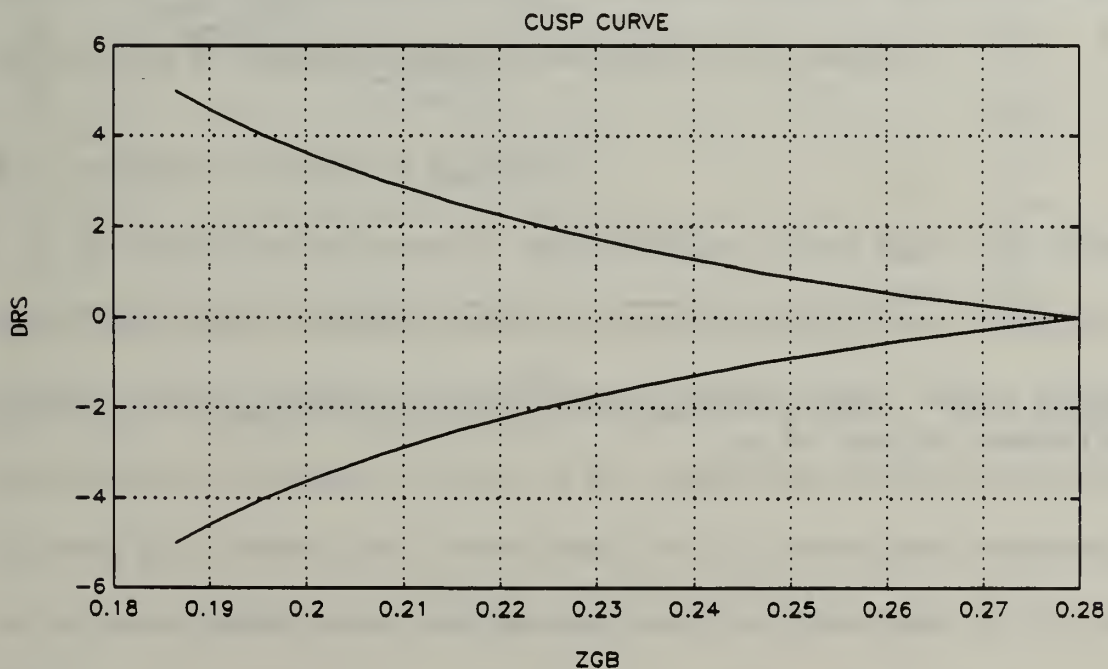


Figure 120. Second Cusp Curve When Continuation Parameter is z_{GB}

As a final study in this chapter, we changed the value of y_G in the first continuation run and tried to get the biased cusp curves for this perturbed case. As a result, we can still see a symmetric bifurcation for this perturbed case for a specific parameter combination. It is clear, however, that this situation is very degenerate and can easily be destroyed for different parameter values. In Figure 121, solid lines correspond to $y_G=0$ feet, dashed lines correspond to $y_G=0.001$ feet, dotted lines correspond to $y_G=0.003$ feet, and dash-dot lines correspond to $y_G=0.005$ feet.

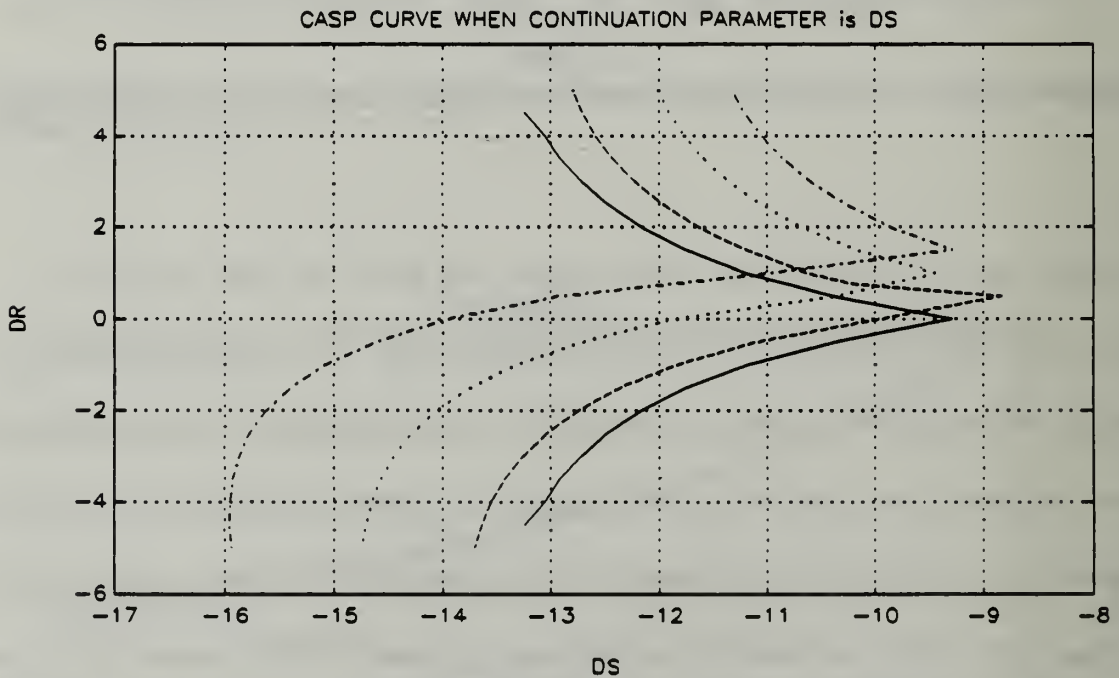


Figure 121. Cusp Curve When Continuation Parameter is δs for Different Values of y_G

IX. DYNAMIC STABILITY ANALYSIS

A. GENERAL

So far, we have performed continuation runs and have obtained steady state solutions of the system. For stability analysis, we chose a representative case and performed the stability analysis for that case. We selected the case when we had x_{GB} as our primary continuation parameter and rudder angle $\delta r = 0$ degrees, and sternplane angle $\delta s = -20$ degrees. To predict dynamic stability, we used the six degree of freedom equations of motion along with the Euler angle rate equations for the derivatives of the angles of pitch and roll ($\dot{\theta}$ and $\dot{\Phi}$). These equations of motion were then linearized around the steady state nominal points computed in the previous chapters. Eigenvalue analysis was then used to compute the stability characteristics of each solution.

B. DYNAMIC STABILITY RESULTS

The stability analysis results are shown in Figure 122 and Figure 123. In these figures, solid lines correspond to stable solutions while dashed lines correspond to unstable solutions. In Figure 122 up to point A, we have a stable solution, which is predominantly forward motion, because we have a high surge velocity and the vehicle is moving in the vertical plane. Between point A and B an out of plane solution exists and the vehicle exhibits motion in the horizontal plane, also. From Figure 123, it is clear

that at point A the sway velocity becomes nonzero. This out of plane solution is stable up to point B, where

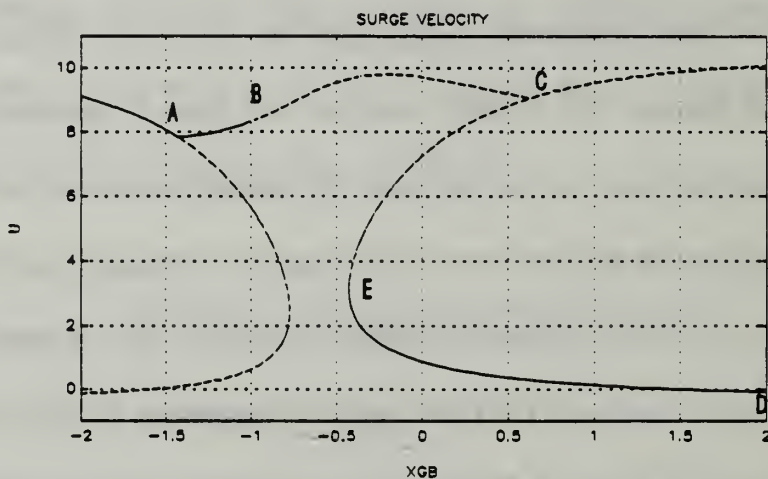


Figure 122. Steady State Surge Velocity

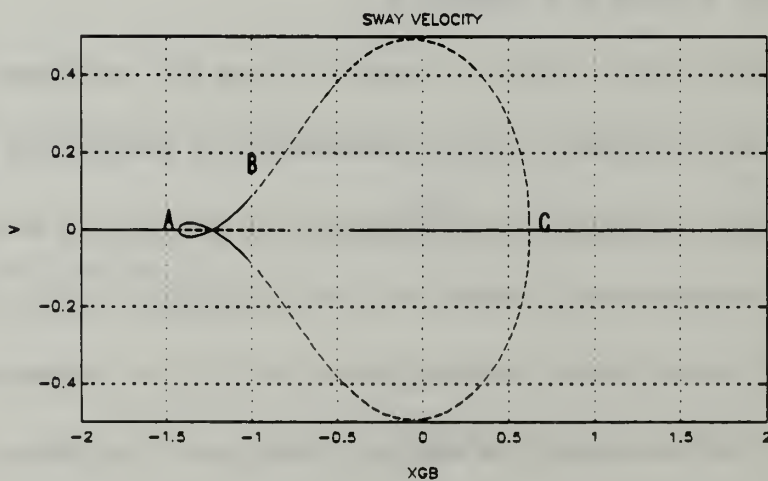


Figure 123. Steady State Sway Velocity

the out of plane motion becomes unstable up to point C, where it disappears. In Figure 122, the motion of the vehicle from point D to E is stable and has a low surge velocity, which means that the motion of the vehicle is predominantly in the upward direction. The vehicle is moving in the vertical plane and there is no horizontal plane motion.

To assess stability of each solution, we used eigenvalue analysis. The eigenvalues that we computed for the following cases are summarized in Table 1.

- Case 1

Sternplane angle $\delta s = -20$ degrees, rudder angle $\delta r = 0$ degrees, $x_{GB} = -2 \% L$, in plane solution.

- Case 2

Sternplane angle $\delta s = -20$ degrees, rudder angle $\delta r = 0$ degrees, $x_{GB} = -1.25 \% L$, out of plane solution.

- Case 3

Sternplane angle $\delta s = -20$ degrees, rudder angle $\delta r = 0$ degrees, $x_{GB} = 0.57 \% L$ point C Figure 122.

- Case 4

Sternplane angle $\delta s = -20$ degrees, rudder angle $\delta r = 0$ degrees, $x_{GB} = 0 \% L$. This is on the stable branch between point D and E in Figure 122.

Table 1 represents the eigenvalues of the system. We can see for case three that there is a pair of complex conjugate eigenvalues with positive real parts. This means that we have a hopf bifurcation prior to that point, and we expect periodic solutions for that part of the solution branch. Since we have a stable solution up to point E in Figure 122,

we don't expect to see this periodic out of plane behavior up to point E. The methods that we used in this work give only stationary solutions, therefore, to see this periodic behavior we used numerical simulation techniques.

TABLE 1

Eigenvalues	Case 1	Case 2	Case 3	Case 4
λ_1	-0.962+0.499	-2.407	-2.591	-0.093+0.522
λ_2	-0.962-0.499	-0.855+0.413	-0.968+0.427	-0.093-0.522
λ_3	-0.068+0.014	-0.855-0.413	-0.968-0.427	-0.067+0.064
λ_4	-0.068-0.014	-0.723	-1.048	-0.067-0.064
λ_5	-0.761	-0.066	-0.314	-0.193+1.531
λ_6	-2.731	-0.018+0.044	-0.129	-0.193-1.531
λ_7	-0.198+0.115	-0.181-0.044	+0.015+0.004	-0.240
λ_8	-0.198-0.115	-0.271	+0.015-0.004	-0.045

C. NUMERICAL INTEGRATION RESULTS

The linearized dynamic response results were verified by simulations using numerical integration of the full six degrees of freedom equations of motion for the swimmer delivery vehicle (SDV). Figures 124 and 125 show a plot of surge velocity (U), and sway velocity (V) versus time respectively for the center of gravity aft of the center of buoyancy case ($x_{GB}=-2$) with a diveplane angle (δ_s) of -20 degrees, and rudder

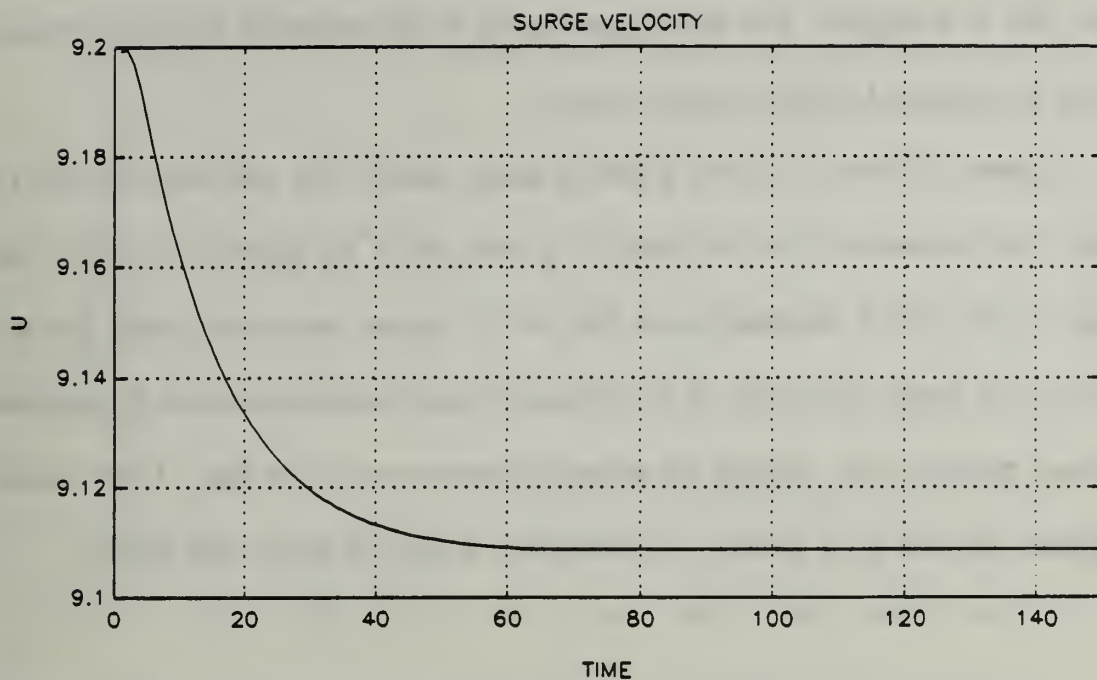


Figure 124. Time History of Surge Velocity for Case 1.

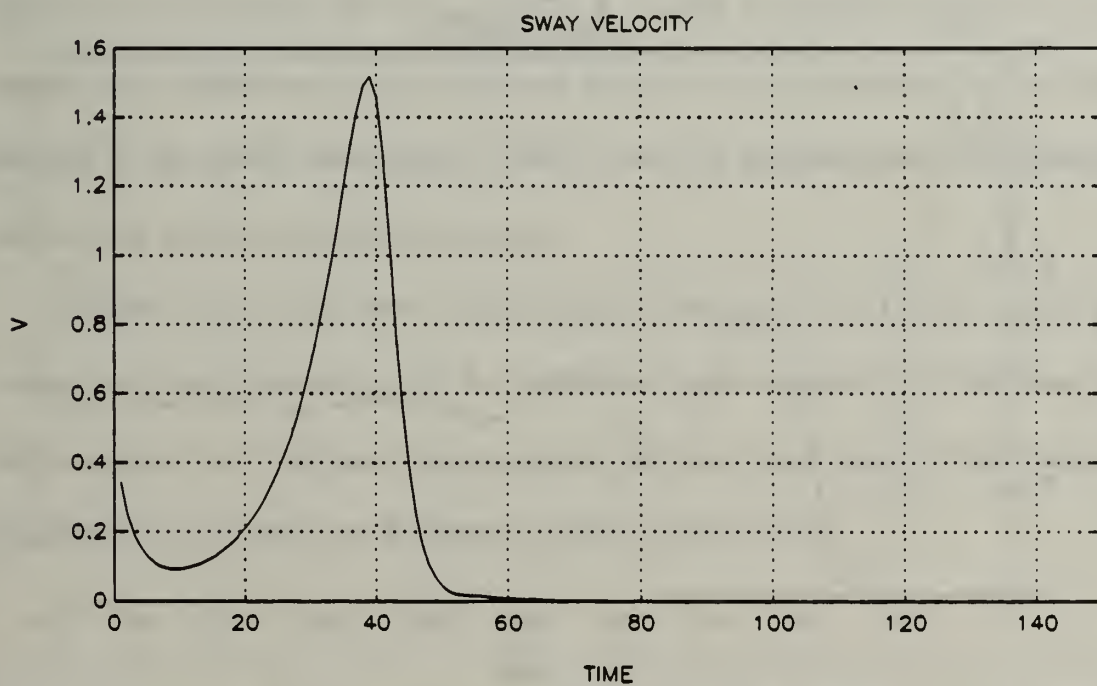


Figure 125. Time History of Sway Velocity for Case 1.

angle (δr) of 0 degrees. The steady state results of the numerical integration method match the linearized dynamic results exactly.

Figures 126 and 127 show a plot of surge velocity (U), and sway velocity (V) versus time respectively for the center of gravity aft of the center of buoyancy case ($x_{GB} = -1.25$) with a diveplane angle (δs) of -20 degrees and rudder angle (δr) of 0 degrees. The steady state results of the numerical integration method match the linearized dynamic results exactly. Vehicle has an out of plane motion in this case. As can be seen in Figure 126, the sway velocity V converged to a nonzero steady state value.

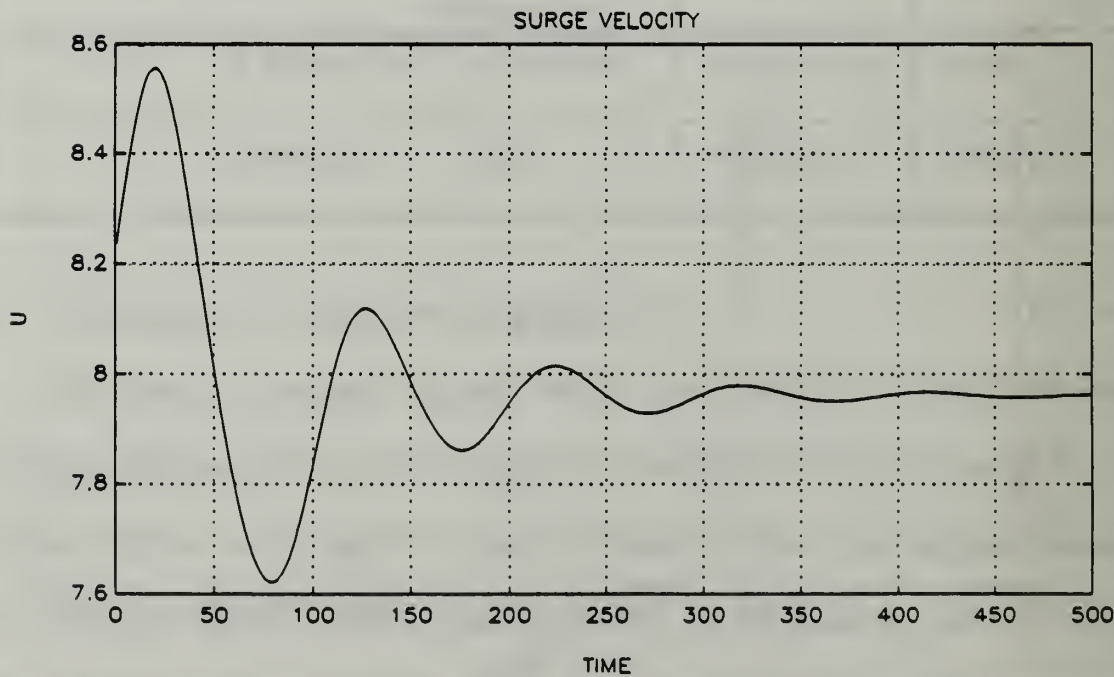


Figure 126. Time History of Surge Velocity for Case 2.

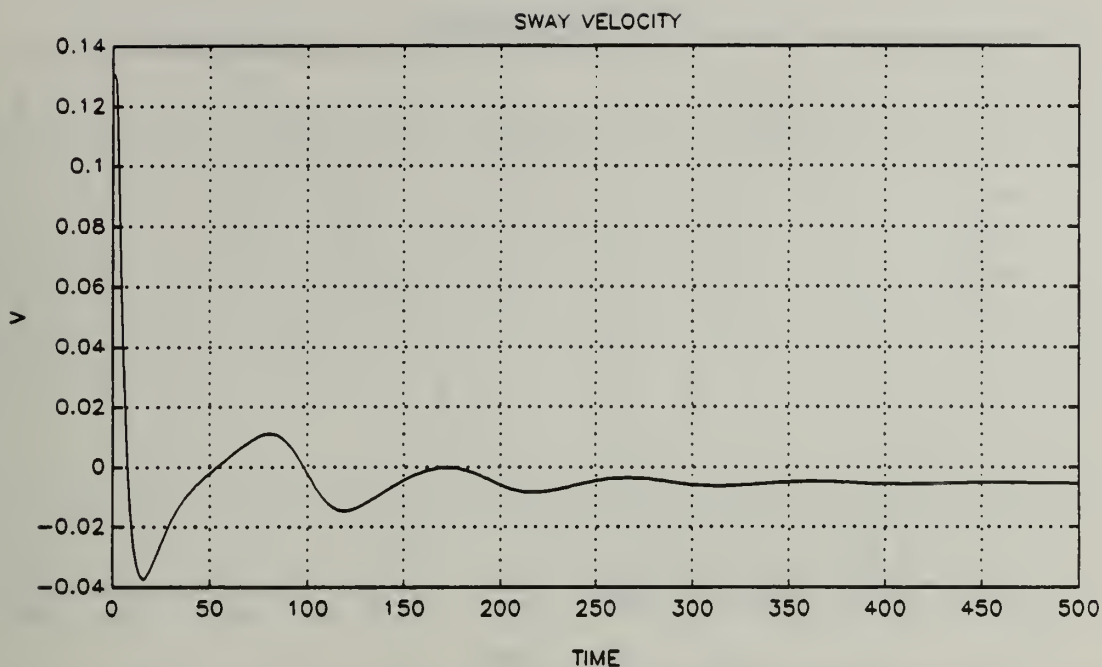


Figure 127. Time History of Sway Velocity for Case 2.

Figures 128 and 129 show a plot of surge velocity (U), and sway velocity (V) versus time respectively for the $x_{GB} = -0.75 \% L$ with a diveplane angle (δ_s) of -20 degrees, and rudder angle (δ_r) of 0 degrees. Here we can see the periodic out of plane behavior of the steady state solution. This is what we expected from the eigenvalue analysis that we did in the previous section.

Figures 130 and 131 show a plot of surge velocity (U), and sway velocity (V) versus time respectively for $x_{GB} = 0 \% L$ with a dive plane angle (δ_s) of -20 degrees, and rudder angle (δ_r) of 0 degrees. And once again, the steady state results of the numerical integration method match the linearized dynamic results exactly.

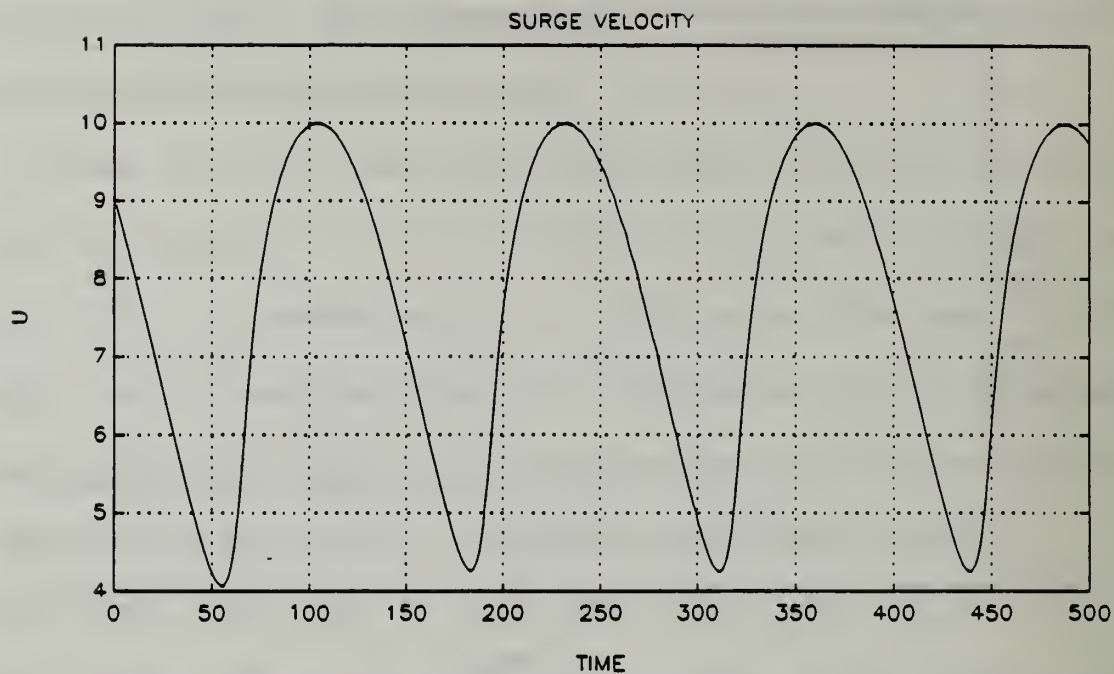


Figure 128. Time History of Surge Velocity for Case 3.

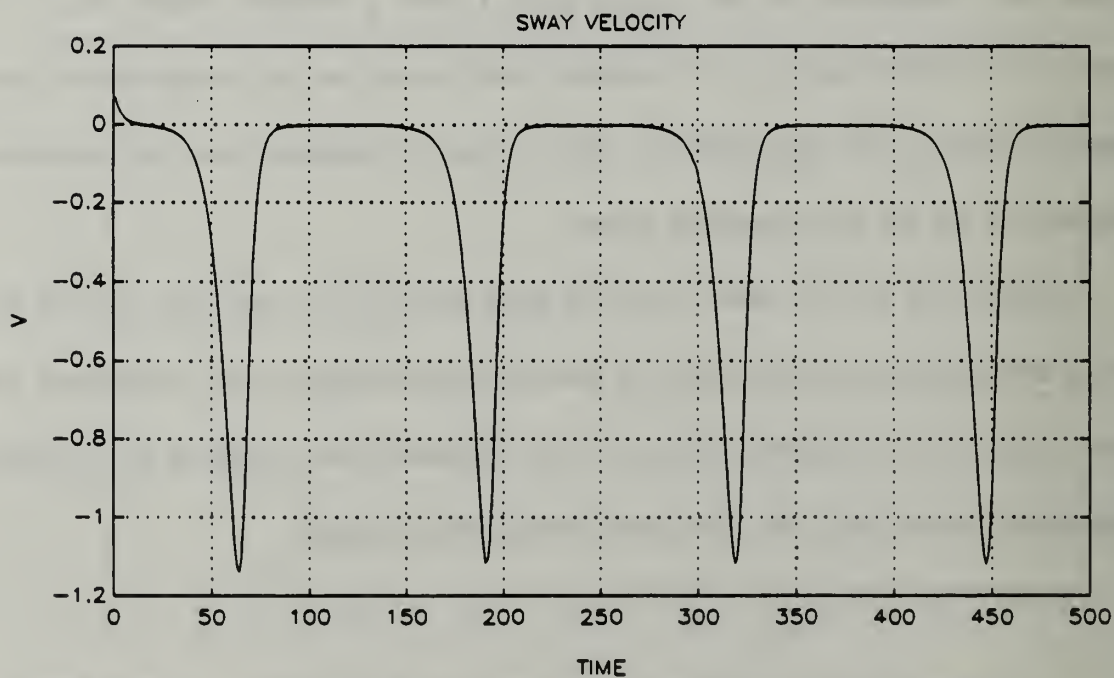


Figure 129. Time History of Sway Velocity for Case 3.

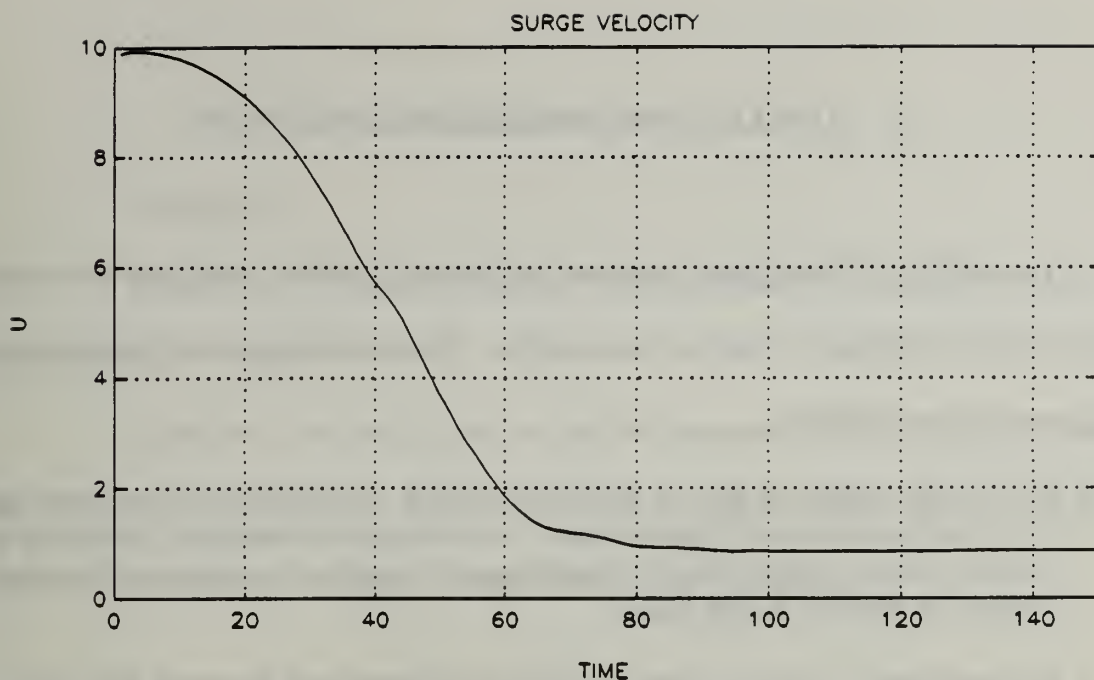


Figure 130. Time History of Surge Velocity for Case 4.

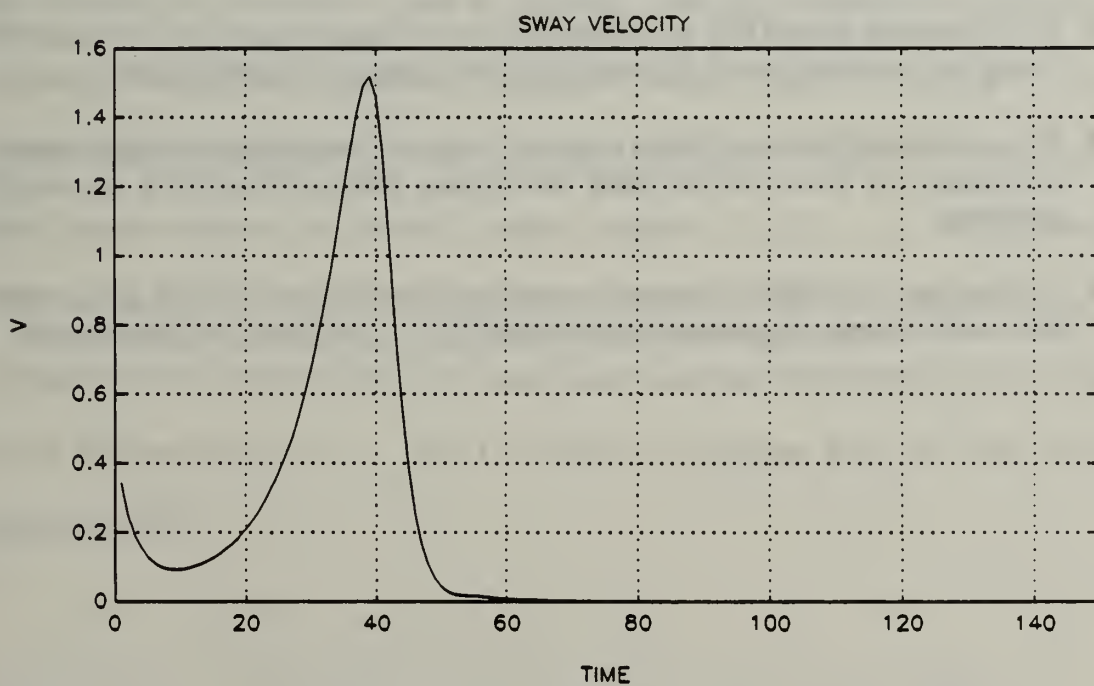


Figure 131. Time History of Sway Velocity for Case 4.

X. CONCLUSIONS AND RECOMMENDATIONS

The problem of steady state response and dynamic stability analysis of submarines in free positive buoyancy ascent has been studied. The main conclusions of this work can be summarized as follows:

- Steady state motion is not, in general, restricted to the vertical plane, there can exist complicated out-of-plane motions. A combination of analytical results for the in plane motions and numerical simulations and continuation was used in order to identify all possible steady states.
- For some steady state solutions there can be pitchfork and hysteresis bifurcations. In fact, it was shown that the pitchfork is the primary mechanism for generating out of plane motions.
- The response of the vehicle can be very sensitive to system parameters. In such a case, any numerical integration results should be viewed with extreme caution.
- The bifurcation graphs that we generated are very useful in order to understand the steady state solution set of the vehicle for any parameter combination.
- As a recommendation for future research, continuation methods should be used to investigate the nature of the Hopf bifurcation, specifically stability of periodic solutions.
- Furthermore, the effect of nonzero propulsion should be investigated. It is possible that small nonzero propulsion forces could affect significantly the bifurcations.

APPENDIX

1. BIFPACK

Bifpack is a program package for calculating bifurcations written in FORTRAN by R. Seydel [Ref. 15]

In this work we used Bifpack to find the bifurcation characteristics of steady state solutions of submersibles. The resulting set of algebraic equations was solved by using PACKA which is devoted to system of nonlinear equations

$$f(y, \lambda) = 0 \quad .$$

Here y and $f(y, \lambda)$ are vectors with n components.

To use Bifpack we did not bother about how to program our original $f(y, \lambda)$ into the enlarged form that is used by Bifpack. This larger frame that embraces the original n scalar algebraic equations by Bifpack is called FRAMEA in Bifpack. To solve our problem we duplicated this frame and gave it different a name. This became our main program and then we entered n scalar formulas $f_1, f_2, f_3, \dots, f_n$ into subroutine FCN. A sample of those two programs is included in the following sections. In conjunction with subroutine FCN, we also wrote functions which compute the integral values in Equations 8, 9, 11, and 12. These are functions FI2, FI3, FI5, FI6 in subroutine FCN.

2. MAIN PROGRAM

```
C                                     SUB2.FOR
PROGRAM MAIN
IMPLICIT DOUBLE PRECISION (A-H,O-Z)

DOUBLE PRECISION EPS

EXTERNAL FCN

C
C  INTERACTIVE WORK WITH THE PACK-A SUBPACKAGE OF
C  BIFPACK
C
COMMON/DATA/DS,DB,DELB,XGB,ZGB,XB,ZB,DRS,DRB
OPEN (11, FILE = 'DATAA1',STATUS='OLD')
OPEN (8 , FILE = 'DIAGRAM',STATUS='OLD')
OPEN (3 , FILE = 'START',STATUS='OLD')
OPEN (1 , FILE = 'OPTIONS',STATUS='OLD')
OPEN (10, FILE = 'FORTRAN.MAT',STATUS='OLD')
OPEN (20, FILE = 'FORTRAN1.MAT',STATUS='OLD')
OPEN (15, FILE='DEN.MAT',STATUS='OLD')
OPEN (16, FILE='DEN1.MAT',STATUS='OLD')

C
C  PROBLEM DEPENDENT VARIABLES:
C  EPS = (REL.) ACCURACY, N = NUMBER OF EQUATIONS
C
  EPS=1.E-5
  N=9
  JACOBI=1

C
C  DATA
C
  PI  =4.0*ATAN(1.0)
  WEIGHT=12000.0
  L=17.425
  DB=0.0*PI/180.0
  DELB=2.0*WEIGHT/100.0
  XGB=-1.0*L/100.0
  ZGB=0.1
  XB=0.0*L/100.0
  ZB=0.0
  DRS=0.0*PI/180.0
```

```

DRB=0.0*PI/180.0
WRITE (*,*) 'WHAT IS DRS'
READ (*,*) DRS
DRS=DRS*PI/180.0
WRITE (*,*) DRS

```

```

WRITE (11,*) 'DS AS PARAMETER ZGB=.1,DB IS 2 PERCENT
              OF WEIGHT'

```

```

WRITE(11,*) '=====',

```

```

C      INITIAL SOLUTION USED FOR THIS RUN

```

```

C      Y(1)=8.6           U
C      Y(2)=0             V
C      Y(3)=-0.25         W
C      Y(4)=0             P
C      Y(5)=0             Q
C      Y(6)=0             R
C      Y(7)=0             PHI
C      Y(8)=0.53          THETA
C      Y(9)=0             PSIDOT

```

```

WRITE (*,*) ' Previous data in DIAGRAM to be purged?'
WRITE (*,*) '  ENTER 0 FOR PURGING, ENTER 1 FOR

```

```

APPENDING:'

```

```

      READ (*,*) NPURGE
      IF (NPURGE.EQ.0) REWIND 8
      IF (NPURGE.EQ.1) THEN
        DO 23 I=1,9999
23      READ (8,'( )',END=29)
29      CONTINUE
      END IF

```

```

CALL INTERA (N,FCN,EPS,JACOBI)
WRITE (*,*) ' PRINT-OUTPUT IN FILE "DATAA1"'
STOP
END

```

```

3.  SUBROUTINE FCN.

```

```

C
SUBROUTINE FCN (TDUMMY,Y,F)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION Y,F,PAR,A,ETACO,ZETACO,PARCO,TDUMMY

```

EOM1.FOR

```

1      DOUBLE PRECISION DB, DR, XG, ZG, YG, XPROP, I2, I3, YB,
2      KPROP, I5, XB, ZB, BOY, I6 ,NPROP, U, V, W,
3      FI2, FI3,FI5,FI6
1      DOUBLE PRECISION WEIGHT, IX, IY, IZ, IXY, IYZ, IXZ, L, RHO,
2      G, A0, CD0, M, XPP, XQQ, XRR, XPR, XUDOT,
3      XWQ, XVP, XVR, XQDS, XQDB, XDRDR, XRDR, XVV,
4      XWW,XVDR,XWDS,XWDB,XDSDS,XDBDB,XRES, YPDOT,
5      YRDOT,YPQ,YQR,YVDOT,YP,YR,YVQ,YWD,YWR,YV,
6      YVW, YDR, YDRB, ZVV, ZDS, ZDB, KPDOT, K VW,
7      MQDOT, MPP, MPR, ZQDOT, ZPP, ZPR, ZRR,
8      ZWDOT,ZQ,ZVP,ZVR,ZW,NRDOT,NPQ,NQR, KRDOT,
9      KPQ, KQR, KVDOT, KP, KR, KVQ, KWP, KWR, KV,
1     MRR, MWDOT, MQ, MVP, MVR, MW, MVV, MDS, MDB,
1     NPDOT,NVDOT, NP, NR, NVQ, NWP, NWR, NV, NVW,
2     NDR, NDRB, XRDRB, XRDRS
      PARAMETER (NDIM=12,NDIML=13)
      DIMENSION Y(NDIML),F(NDIML)
C
C   FOLLOWING THREE STATEMENTS ARE FOR CONTINUATION AN
C   BRANCH SWITCHING  PURPOSES:
C
COMMON/DATA/DS,DB,DELB,XGB,ZGB,XB,ZB,DRS,DRB
COMMON /CONTRO/ ETACO,ZETACO,PARCO,NCO,N1CO,N2CO,
1     KCO,IND2CO,NFLAG
      PAR=Y(N1CO)
      F(N1CO)=Y(KCO)-ZETACO*Y(IND2CO)-ETACO
C
C   GEOMETRIC PROPERTIES
C
      WEIGHT=12000.0
      IX=1760.0
      IY=9450.0
      IZ=10700.0
      IXY=0.0
      IYZ=0.0
      IXZ=0.0
      L=17.425
      RHO=1.94
      G=32.2
      YG=0.1
      YB=0.0
      A0=2.0
      CD0=0.0057

```

MASS=WEIGHT/G

C
C
C

SURGE HYDRODYNAMIC COEFFICIENTS

XPP = $7.030E-03*0.5*RHO*L^{**4}$
XQQ = $-1.470E-02*0.5*RHO*L^{**4}$
XRR = $4.010E-03*0.5*RHO*L^{**4}$
XPR = $7.640E-04*0.5*RHO*L^{**4}$
XUDOT = $-7.580E-03*0.5*RHO*L^{**3}$
XWQ = $-1.920E-01*0.5*RHO*L^{**3}$
XVP = $-3.240E-03*0.5*RHO*L^{**3}$
XVR = $1.890E-02*0.5*RHO*L^{**3}$
XQDS = $2.610E-02*0.5*RHO*L^{**3}$
XQDB = $-2.600E-03*0.5*RHO*L^{**3}$
XRDR = $-8.180E-04*0.5*RHO*L^{**3}$
XVV = $5.290E-02*0.5*RHO*L^{**2}$
XWW = $1.710E-01*0.5*RHO*L^{**2}$
XVDR = $1.730E-03*0.5*RHO*L^{**2}$
XWDS = $4.600E-02*0.5*RHO*L^{**2}$
XWDB = $9.660E-03*0.5*RHO*L^{**2}$
XDSDS = $-1.160E-02*0.5*RHO*L^{**2}$
XDBDB = $-8.070E-03*0.5*RHO*L^{**2}$
XDRDR = $-1.010E-02*0.5*RHO*L^{**2}$
XRES = $CD0*0.5*RHO*L^{**2}$
XPROP = XRES*ALPHA**2
XRDRB=XRDR
XRDRS=XRDR
XVDRS=XVDR
XDRB=XVDR

C
C
C

SWAY HYDRODYNAMIC COEFFICIENTS

YPDOT = $1.270E-04*0.5*RHO*L^{**4}$
YRDOT = $1.240E-03*0.5*RHO*L^{**4}$
YPQ = $4.125E-03*0.5*RHO*L^{**4}$
YQR = $-6.510E-03*0.5*RHO*L^{**4}$
YVDOT = $-5.550E-02*0.5*RHO*L^{**3}$
YP = $3.055E-03*0.5*RHO*L^{**3}$
YR = $2.970E-02*0.5*RHO*L^{**3}$
YVQ = $2.360E-02*0.5*RHO*L^{**3}$
YWP = $2.350E-01*0.5*RHO*L^{**3}$
YWR = $-1.880E-02*0.5*RHO*L^{**3}$
YV = $-9.310E-02*0.5*RHO*L^{**2}$

YVW = $6.840E-02 * 0.5 * \text{RHO} * L^{**2}$
 YDRS = $+2.270E-02 * 0.5 * \text{RHO} * L^{**2}$
 YDRB = $+2.270E-02 * 0.5 * \text{RHO} * L^{**2}$

C
C
C

HEAVE HYDRODYNAMIC COEFFICIENTS

ZQDOT = $-6.810E-03 * 0.5 * \text{RHO} * L^{**4}$
 ZPP = $1.270E-04 * 0.5 * \text{RHO} * L^{**4}$
 ZPR = $6.670E-03 * 0.5 * \text{RHO} * L^{**4}$
 ZRR = $-7.350E-03 * 0.5 * \text{RHO} * L^{**4}$
 ZWDOT = $-2.430E-01 * 0.5 * \text{RHO} * L^{**3}$
 ZQ = $-1.350E-01 * 0.5 * \text{RHO} * L^{**3}$
 ZVP = $-4.810E-02 * 0.5 * \text{RHO} * L^{**3}$
 ZVR = $4.550E-02 * 0.5 * \text{RHO} * L^{**3}$
 ZW = $-3.020E-01 * 0.5 * \text{RHO} * L^{**2}$
 ZVV = $-6.840E-02 * 0.5 * \text{RHO} * L^{**2}$
 ZDS = $-2.270E-02 * 0.5 * \text{RHO} * L^{**2}$
 ZDB = $-2.270E-02 * 0.5 * \text{RHO} * L^{**2}$

C
C
C

ROLL HYDRODYNAMIC COEFFICIENTS

KPDOT = $-1.010E-03 * 0.5 * \text{RHO} * L^{**5}$
 KRDOT = $-3.370E-05 * 0.5 * \text{RHO} * L^{**5}$
 KPQ = $-6.930E-05 * 0.5 * \text{RHO} * L^{**5}$
 KQR = $1.680E-02 * 0.5 * \text{RHO} * L^{**5}$
 KVDOT = $1.270E-04 * 0.5 * \text{RHO} * L^{**4}$
 KP = $-1.100E-02 * 0.5 * \text{RHO} * L^{**4}$
 KR = $-8.410E-04 * 0.5 * \text{RHO} * L^{**4}$
 KVQ = $-5.115E-03 * 0.5 * \text{RHO} * L^{**4}$
 KWP = $-1.270E-04 * 0.5 * \text{RHO} * L^{**4}$
 KWR = $1.390E-02 * 0.5 * \text{RHO} * L^{**4}$
 KV = $3.055E-03 * 0.5 * \text{RHO} * L^{**3}$
 KVW = $-1.870E-01 * 0.5 * \text{RHO} * L^{**3}$

C
C
C

PITCH HYDRODYNAMIC COEFFICIENTS

MQDOT = $-1.680E-02 * 0.5 * \text{RHO} * L^{**5}$
 MPP = $5.260E-05 * 0.5 * \text{RHO} * L^{**5}$
 MPR = $5.040E-03 * 0.5 * \text{RHO} * L^{**5}$
 MRR = $-2.860E-03 * 0.5 * \text{RHO} * L^{**5}$
 MWDOT = $-6.810E-02 * 0.5 * \text{RHO} * L^{**4}$
 MQ = $-6.860E-02 * 0.5 * \text{RHO} * L^{**4}$
 MVP = $1.180E-03 * 0.5 * \text{RHO} * L^{**4}$

$MVR = 1.730E-02 * 0.5 * RHO * L^{**4}$
 $MW = 9.860E-02 * 0.5 * RHO * L^{**3}$
 $MVV = -2.510E-02 * 0.5 * RHO * L^{**3}$
 $MDS = -1.113E-02 * 0.5 * RHO * L^{**3}$
 $MDB = 1.113E-02 * 0.5 * RHO * L^{**3}$

C

C YAW HYDRODYNAMIC COEFFICIENTS

C

$NPDOT = -3.370E-05 * 0.5 * RHO * L^{**5}$
 $NRDOT = -3.400E-03 * 0.5 * RHO * L^{**5}$
 $NPQ = -2.110E-02 * 0.5 * RHO * L^{**5}$
 $NQR = 2.750E-03 * 0.5 * RHO * L^{**5}$
 $NVDOT = 1.240E-03 * 0.5 * RHO * L^{**4}$
 $NP = -8.405E-04 * 0.5 * RHO * L^{**4}$
 $NR = -1.640E-02 * 0.5 * RHO * L^{**4}$
 $NVQ = -9.990E-03 * 0.5 * RHO * L^{**4}$
 $NWP = -1.750E-02 * 0.5 * RHO * L^{**4}$
 $NWR = 7.350E-03 * 0.5 * RHO * L^{**4}$
 $NV = -7.420E-03 * 0.5 * RHO * L^{**3}$
 $NVW = -2.670E-02 * 0.5 * RHO * L^{**3}$
 $NDRS = -1.113E-02 * 0.5 * RHO * L^{**3}$
 $NDRB = +1.113E-02 * 0.5 * RHO * L^{**3}$

C

C VARIABLE INITIALIZATION

C

$RPM = 0.0$
 $BOY = WEIGHT + DELB$
 $XG = XB + XGB$
 $ZG = ZB + ZGB$
 $ALPHA = 0.0$

C

C TDUMMY IS DUMMY

C THIS ROUTINE EVALUATES THE PROBLEM DEFINING FUNCTION
 C INPUT IS $Y(1), \dots, Y(N)$, AND $PAR = \text{PARAMETER NOW}, F(1), \dots$
 C $, F(N)$ OF THE FUNCTION OF RIGHT HAND SIDE OF $F(Y, PAR)$ ARE
 C ENTERED: (SIX DEGREE FREEDOM EQUATION OF MOTION OF C
 C SUBMARINE)
 C

$U = Y(1)$
 $V = Y(2)$
 $W = Y(3)$
 $P = Y(4)$
 $Q = Y(5)$

R=Y(6)
 PHI=Y(7)
 THETA=Y(8)
 PSIDOT=Y(9)
 I2=FI2(V,W,Q,R)
 I3=FI3(V,W,Q,R)
 I5=FI5(V,W,Q,R)
 I6=FI6(V,W,Q,R)
 SWAY=I2
 HEAVE=I3
 PITCH=I5
 YAW=I6

PARAMETER IS DS FOR THIS RUN

DS=PAR

SURGE FORCE

F(1) = MASS*V*R-MASS*W*Q+MASS*XG*Q**2+MASS*XG*R**2-
 & MASS*YG*P*Q-MASS*ZG*P*R+XPP*P**2+XQQ*Q**2+XRR*R**2+
 & XPR*P*R+XWQ*W*Q+XVP*V*P+XVR*V*R+U*Q*
 & (XQDS*DS+XQDB*DB)+
 & U*R*(XRDRS*DRS+XRDRB*DRB)+XVV*V**2+XWW*W**2+U*V*
 & (XVDRS*DRS+XDRB*DRB)+U*W*(XWDS*DS+XWDB*DB)+
 & (XDSDS*DS**2+XDBDB*DB**2+XDRDR*(DRS**2+DRB**2))*U**2-
 & (WEIGHT-BOY) * DSIN(THETA)+XPROP*RPM*RPM-XRES*U*U

SWAY FORCE

F(2) = -MASS*U*R-MASS*XG*P*Q+MASS*YG*R**2-MASS*ZG*Q*R+
 & YPQ*P*Q+YQR*Q*R+YP*U*P+YR*U*R+YVQ*V*Q+YWP*W*P+
 & YWR*W*R+YV*U*V+YVW*V*W+YDRS*U**2*DRS+YDRB*U**2*
 & DRB+ (WEIGHT-BOY)* DCOS(THETA)*DSIN(PHI)+MASS*W*P+
 & MASS*YG*P**2+SWAY

HEAVE FORCE

F(3) = MASS*U*Q-MASS*V*P-MASS*XG*P*R-MASS*YG*Q*R+
 & MASS*ZG*P**2+MASS*ZG*Q**2+ZPP*P**2+ZPR*P*R+
 & ZRR*R**2+ZQ*U*Q+ZVP*V*P+ZVR*V*R+ZW*U*W+ZVV*V**2+
 & HEAVE+ U**2*(ZDS*DS+ZDB*DB)+(WEIGHT-BOY)*
 & DCOS(THETA)* DCOS(PHI)

C
C ROLL MOMENT
C

F(4) = -IZ*Q*R+IY*Q*R-IXY*P*R+IYZ*Q**2-IYZ*R**2+IXZ*P*Q+
& MASS*YG*U*Q-MASS*YG*V*P-MASS*ZG*W*P+KPQ*P*Q+
& KQR*Q*R+ KP*U*P+KR*U*R+KVQ*V*Q+KWP*W*P+KWR*
& W*R+KV*U*V+ KVV*V*W+
& (YG*WEIGHT-YB*BOY)*DCOS(THETA)*DCOS(PHI)-(ZG*WEIGHT-
& ZB*BOY)*DCOS(THETA)*DSIN(PHI)+MASS*ZG*U*R

C
C PITCH MOMENT
C

F(5) = -IX*P*R+IZ*P*R+IXY*Q*R-IYZ*P*Q-IXZ*P**2+IXZ*R**2-
& MASS*XG*U*Q+MASS*XG*V*P+MASS*ZG*V*R-MASS*ZG*W*Q+
& MPP*P**2+MPR*P*R+MRR*R**2+MQ*U*Q+MVP*V*P+MVR*V*R
& +MW*U*W+MVV*V**2+U**2*(MDS*DS+MDB*DB)-(XG*WEIGHT-
& XB*BOY)*DCOS(THETA)*DCOS(PHI)-
& (ZG*WEIGHT-ZB*BOY)*DSIN(THETA)+PITCH

C
C YAW MOMENT
C

F(6) = -IY*P*Q+IX*P*Q+IXY*P**2-IXY*Q**2+IYZ*P*R-IXZ*Q*R-
& MASS*XG*U*R+MASS*XG*W*P-MASS*YG*V*R+MASS*YG*W*Q+
& NPQ*P*Q+NQR*Q*R+NP*U*P+NR*U*R+NVQ*V*Q+NWP*W*P+
& NWR*W*R+NV*U*V+ NVW*V*W+NDRS*U**2*DRS+NDRB*
& U**2*DRB+(XG*WEIGHT-XB*BOY)*DCOS(THETA)*DSIN(PHI)+
& (YG*WEIGHT-YB*BOY)*DSIN(THETA)+YAW

C
C KINEMATIC RELATIONS
C

F(7) = P+PSIDOT*DSIN(THETA)

F(8) = Q-PSIDOT*DSIN(PHI)*DCOS(THETA)

F(9) = R-PSIDOT*DCOS(PHI)*DCOS(THETA)

RETURN

END

4. SAMPLE RUN

MARS \$ run sub2

WHAT IS DRS

0.

0.0000000000000000E+00

Previous data in DIAGRAM to be purged?

ENTER 0 FOR PURGING, ENTER 1 FOR APPENDING:

0

THIS IS THE MENU YOU CAN CHOOSE FROM:

- 0 : ENTERING STARTING DATA
- 1 : CONTINUATION: NO RESTRICTIONS
- 2 : CONTINUATION: SETTING OF OPTIONS, THEN RUN
- 3 : CONTINUATION, FOLLOWING PREVIOUS OPTIONS
- 1 : ENDING THIS SESSION

ENTER CHOICE of MODUS:

0

ENTER COMPONENT Y(1):

4.2

ENTER COMPONENT Y(2):

0

ENTER COMPONENT Y(3):

-.7

ENTER COMPONENT Y(4):

0

ENTER COMPONENT Y(5):

0

ENTER COMPONENT Y(6):

0

ENTER COMPONENT Y(7):

0

ENTER COMPONENT Y(8):

.1745

ENTER COMPONENT Y(9):

0

ENTER PARAMETER:

.345

THIS IS THE MENU YOU CAN CHOOSE FROM:

- 0 : ENTERING STARTING DATA
- 1 : CONTINUATION: NO RESTRICTIONS
- 2 : CONTINUATION: SETTING OF OPTIONS, THEN RUN
- 3 : CONTINUATION, FOLLOWING PREVIOUS OPTIONS
- 1 : ENDING THIS SESSION

ENTER CHOICE of MODUS:

2

CURRENT STARTING DATA AT PARAMETER: 0.3450000000000000

THESE ARE THE CURRENT OPTIONS:

No: 1 , step length : -0.17406E-01

No: 2 , index of fixed component : -1

No: 3 , level of continuation : 0
No: 4 , maximum number of continuation steps : 200
No: 5 , step bounds: Predictor:0.100E+00 , Param.:0.100E+00 , rel.change:0.10
No: 6 , stability analysis (yes=1, no=0) : 0
No: 7 , window : -0.35000E+00 .lt. parameter .lt. 0.35000E+00
No: 8 , target point: no final value is set.

ENTER Number OF OPTION YOU WANT TO CHANGE, OR
ENTER 0 TO RUN THE CONTINUATION, OR
ENTER -1 TO RETURN TO MENU:

1
ENTER INITIAL STEPSIZE (IN REAL):
-.005

ENTER Number OF OPTION YOU WANT TO CHANGE, OR
ENTER 0 TO RUN THE CONTINUATION, OR
ENTER -1 TO RETURN TO MENU:

2
CHOOSE INDEX FOR INITIAL STEP:
ENTER 0 FOR CONTIN. WITH RESPECT TO PARAMETER,
ENTER K FOR FIXING K-TH COMPONENT:

0
ENTER Number OF OPTION YOU WANT TO CHANGE, OR
ENTER 0 TO RUN THE CONTINUATION, OR
ENTER -1 TO RETURN TO MENU:

3
CHOOSE AMONG THREE LEVELS OF STEP CONTROL:
ENTER 2 FOR FREELY VARIABLE STEPS,
1 FOR VARIABLE STEP BUT FIXED INDEX,
0 FOR BOTH STEPLENGTH AND INDEX FIXED:

0
ENTER Number OF OPTION YOU WANT TO CHANGE, OR
ENTER 0 TO RUN THE CONTINUATION, OR
ENTER -1 TO RETURN TO MENU:

4
Enter maximum NUMBER OF CONTINUATION STEPS:
100

ENTER Number OF OPTION YOU WANT TO CHANGE, OR
ENTER 0 TO RUN THE CONTINUATION, OR
ENTER -1 TO RETURN TO MENU:

-1
THIS IS THE MENU YOU CAN CHOOSE FROM:

0 : ENTERING STARTING DATA
1 : CONTINUATION: NO RESTRICTIONS

- 2 : CONTINUATION: SETTING OF OPTIONS, THEN RUN
- 3 : CONTINUATION, FOLLOWING PREVIOUS OPTIONS
- 1 : ENDING THIS SESSION

ENTER CHOICE of MODUS:

2

CURRENT STARTING DATA AT PARAMETER: 0.3450000000000000

THESE ARE THE CURRENT OPTIONS:

No: 1 , step length : -0.50000E-02

No: 2 , index of fixed component : 0

No: 3 , level of continuation : 0

No: 4 , maximum number of continuation steps : 100

No: 5 , step bounds: Predictor:0.100E+00 , Param.:0.100E+00 , rel.change:0.10

No: 6 , stability analysis (yes=1, no=0) : 0

No: 7 , window : -0.35000E+00 .lt. parameter .lt. 0.35000E+00

No: 8 , target point: no final value is set.

ENTER Number OF OPTION YOU WANT TO CHANGE, OR

ENTER 0 TO RUN THE CONTINUATION, OR

ENTER -1 TO RETURN TO MENU:

0

LIST OF REFERENCES

1. Clayton, B.R and Bishop, R.E.D. Mechanics of Marine Vehicles, Gulf Publishing Company, Houston, 1982.
2. Roddy, R.F, Investigation of the stability and control characteristics of several configurations of the DARPA SUBOFF model (DRTC model 5470) from captive-model experiments, David Taylor Research Center, Report DRTC/SHD-1298-08, 1990.
3. Falzarano, J.M., Shaw, S.W, and Troesch, A.W, "Application of Global Methods for Analyzing Dynamical Systems to Ship Rolling Motion and Capsizing", Journal of Bifurcation and Chaos, 2, 1, 1992.
4. Taz Ul Mulk, M. and Falzarano, J., "Complete Six Degrees of Freedom Nonlinear Ship Rolling Motion", Ocean Engineering, Submitted for publication, 1993.
5. Sarpkaya, T., "Brief Reviews of Some Time-Dependent Flows", Journal of Fluids Engineering, Vol. 114/283, September 1992.
6. Lugt, H.J., "Numerical Modeling of Vortex Flows in Ship Hydrodynamics -A Review", Proceedings of the Third International Conference on Numerical Ship Hydrodynamics, Vol. 1, 1981.
7. Booth, T.B., "Stability of Buoyant Underwater Vehicles Part I. Predominantly Forward Motion", International Shipbuilding Progress, 24, 279, 1977.
8. Booth, T.B., "Stability of Buoyant Underwater Vehicles Part II. Near Vertical Ascent", International Shipbuilding Progress, 24, 280, 1977.
9. Papoulias, F.A., and McKinley, B.D., "Inverted Pendulum Stabilization of Submarines in Free Positive Buoyancy Ascent:", Journal of Ship Research, In press, 1993.
10. Golubitsky, M. and Schaeffer, D.G., Singularities and Groups in Bifurcation Theory I, Springer-Verlag, 1985.

11. Guckenheimer, J. and Holmes P., Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields, Applied Mathematical Sciences 42, Springer-Verlag, 1983.
12. Seydel, R., From Equilibrium to Chaos Practical Stability and Bifurcation Analysis, Elsevier, 1988.
13. Smith, N.S., Crane, J.W., and Summey, D.C., SDV Simulator Hydrodynamic Coefficients, Report NCSC-TM231-78, Naval Coastal Systems Center, 1978.
14. McKinley B.D., "Dynamic Stability of Positively Buoyant Submersibles: Vertical Plane Solutions", Master's Thesis, Naval Postgraduate School, Monterey, California, December 1991.
15. Seydel, R., "BIFFPACK: A Program Package for Calculating Bifurcations", State University of New York at Buffalo, 1985.

INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Technical Information Center Cameron Station Alexandria VA 22304-6145	2
2. Library, Code 052 Naval Postgraduate School Monterey CA 93943-5002	2
3. Deniz Harp Okulu Tuzla Istanbul, TURKEY	2
4. Golcuk Tersanesi Komutanligi Golcuk Kocaeli, TURKEY	2
5. Taskizak Tersanesi Komutanligi Kasimpasa Istanbul, TURKEY	2
6. Deniz Kuvvetleri Komutanligi Personel Egitim Daire Baskanligi Bakanliklar Ankara, TURKEY	1
7. Professor Fotis A. Papoulias, Code ME/Pa Department of Mechanical Engineering Naval Postgraduate School Monterey, CA 93943-5100	1
8. Department Chairman, Code ME/Kk Department Of Mechanical Engineering Naval Postgraduate School Monterey, CA 93943-5100	1

9. Naval Engineering Curricular Office (Code 34) 1
Naval Postgraduate School
Monterey, CA 93943-5100
10. Ibrahim AYDIN 1
Mobil Karsisi Serdar Apt.
No: 241/4 Ciftlikkoy
Yalova Istanbul/TURKEY

BUDLEY KNOX LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY CA 93946-5101



3 2768 00311323 4